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A

PRACTICAL AND THEORETICAL

ARITHMETIC:

DESIGNED FOR

THE USE OF SCHOOLS AND ACADEMIES.

BY

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PREFACE.

Arithmetic is a science so universally studied, and of so much practical importance, that any attempt to prepare a treatise on this subject which shall be well adapted to the wants of the student, is worthy of encouragement. It is, indeed, a fortunate circumstance for the interests of education, that so large a number of text books in every department of science and literature can be found from which the intelligent teacher can select whatever he deems best suited to the purposes of instruction. Entertaining the belief that there is still a chance to prepare a treatise on Arithmetic which may contribute something to the means of education, we have made an effort to make that contribution, and the present volume is the result of our labors.

Believing that conciseness is a great merit in a text book, we have endeavored to state the principles of the science, to point out their application, and to give all necessary explanations, with brevity. At the same time we have made an effort to present everything with clearness and fulness, and to give the pupil all the necessary aid.

In most of our treatises on Arithmetic, questions on the rules and principles of the science may be found at the bottom of the page, or at the close of the book. We are aware that there are many teachers of ability who conduct their recitations by means of questions, but our experience in teaching has induced us to think that the pupil would receive much greater mental discipline, if he were obliged to give a thorough analysis of his lesson without the aid of any leading questions on the part of the *teacher*.

If a recitation is to be conducted by means of questions, we are of the opinion that it would be better to let the pupils question each other or their teacher; but in this case, it is obvious that they should have such a thorough acquaintance with the subject, that they may propose the proper questions, and in their proper order, without having any book in their hands. If, however, the teacher wishes to propose the questions, it is to be presumed that his familiarity with the subject is such that he will not be under the very disagreeable necessity of letting his eye glance at the bottom of the page, in order that he may know what question he ought to propose. It often happens that he has use for both of his eyes in another direction. We cannot, therefore, see any good reason for inserting these questions, and they are consequently omitted in this work.

A special effort has been made to insert such examples in this work as would interest the pupil, and give him the greatest amount of mental discipline. Much of the merit of an Arithmetic depends upon the character of its examples, since the young can often comprehend a principle when it is applied in the solution of an appropriate example, which they could not understand if it were stated in an abstract manner.

We have not room for enumerating all the peculiarities of the work, and it is unnecessary to dwell on them. We will only direct the attention of teachers to the chapters on analysis, duodecimals, and evolution. We have spared no pains nor study to prepare a useful and thorough treatise on the subject, and we respectfully ask those who are interested in educational matters, to give it an examination.

HOMER, October 10, 1853.

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ARITHMETIC.

CHAPTER I.

DEFINITIONS.

1. *Quantity* is any thing which may be increased or diminished.

2. A *unit* is one single thing; as one chair, one apple.

3. *Number* is a term used to denote a unit or an assemblage of units. Hence, numbers may be employed to represent quantities.

4. A *concrete number* is an expression for a collection of units of a *particular kind*. *Eight dollars* is a concrete number.

5. An *abstract number* is an expression for a collection of units of no particular kind. *Eight* and *Five* are abstract numbers.

6. *Notation* is the art of expressing numbers by certain characters. The characters which are generally employed are called figures. These figures are, 1 one, 2 two, 3 three, 4 four, 5 five, 6 six, 7 seven, 8 eight, 9 nine, 0 cipher or nothing. By combining these figures in different ways, they can be made to express any number.

7. *Numeration* is the art of reading numbers which are expressed by figures, or by other characters.

8. The *unit of measure or comparison*, is one of the individ-

ual things which make up a concrete number. In measuring distances, the unit of measure is a *line;* in measuring solids, the unit of measure is a solid; in measuring surfaces, the unit of measure is a surface, and this surface is a *square* surface. The unit of measure may be of any convenient magnitude. In the number 5 *inches*, the unit of measure is *a line one inch long.* In the number 5 *miles*, the unit of measure is *a line one mile long.*

9. The symbol + denotes addition; as, $5+3$, or 5 added to 3.

10. The symbol − denotes subtraction; as, $5-3$, or 3 subtracted from 5.

11. The symbol × denotes multiplication; as, 5×3, or 5 multiplied by 3.

12. The symbol ÷ denotes division; as, $6\div2$, or 6 divided by 2.

13. The symbol = denotes equality; as $5=5$, or 5 equals 5.

14. A *Problem* is a question proposed which requires something to be ascertained.

CHAPTER II.

NOTATION AND NUMERATION.

NOTATION.

1. Notation presents for solution, the following

PROBLEM.

It is required to express by means of the ten figures, any number.

If the number does not exceed nine, it may be represented by one of the nine figures, 1, 2, 3, 4, 5, 6, 7, 8, 9; but if it exceeds nine, we must seek to represent it by a part or all of the ten figures. For this purpose, let us agree or establish the convention, that, when figures are arranged in a horizontal line, the first, or right-hand figure shall represent simply *units*, the second, *tens*, the third, *hundreds*, the fourth, *thousands*, the fifth, *tens of thousands*, the sixth, *hundreds of thousands*, the seventh, *millions*, and so on. Hence, by this method of notation, any number may be represented by means of figures.

For example, let it be required to express, by means of figures, the number *two hundred and seventy-five.*

In this number, there are 5 units, 7 tens, and 2 hundreds. Hence, the expression, for the number is, 275.

As another example, let it be required to express by means of figures, the number *four hundred and seven.*

In this number, there are 7 units, 0 tens, and 4 hundreds.

Hence, the expression, for the number is 407. The cipher is used to denote that there are no tens.

The first nine figures are called *significant figures*, and they all represent units, but these units are not of the same value, when these figures are employed to express any number. Thus, in the number 483, *one* of the 8 units is equal to *ten* times one of the 3 units; and *one* of the 4 units is equal to *ten* times one of the 8 units, or *one hundred* times one of the three units. Hence, it is a law of this system of notation, that *the value of any figure is increased ten times, by removing it so that it may stand in the next left-hand place.* Thus, the value represented by the figure 7, is increased ten times, if we cause it to occupy the next left-hand place, by placing a cipher at its right hand. It then becomes 70, or seventy, which is ten times 7.

It will be noticed that figures have two values, namely, a *simple* and a *local* value. The simple value of a figure is the value which it represents when it stands alone, or occupies the first or left-hand place. The *local* value of a figure is the value which it has, when connected with other figures. When a figure occupies the first or right-hand place, in a number, the units which it expresses are called units of the *first order;* when it occupies the second place, its units are called units of the second order, and so on.

The character 0 is adopted into this system of notation, so that each figure in a number may stand in the place denoted by the order of units which it expresses. For example, in the number 400, the figure 4, which expresses units of the *third* order, retains the place denoted by this order of units, by means of the two ciphers. The other nine figures are called significant figures.

The system of notation which we have adopted, is called the

decimal system, for the reason that *ten* units of any order make *one* of the next higher order, and that *ten* figures must be employed, in order to express all numbers. The number *ten* is called the base of the system.

By the aid of the following table, the pupil will find no difficulty in writing any proposed number. In the table, the names of the different orders of units are given, from the first order of Trillions. The names of the following orders are *Tens of Trillions*, *Hundreds of Trillions*, *Quadrillions*, &c.

TABLE.

&c.	Trillions.	Hundreds of Billions.	Tens of Billions.	Billions.	Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.
	3	2	1	0	9	8	7	6	5	4	3	2	1

2. When we seek to form an idea of the size or magnitude of a quantity by comparing it with the unit of measure, it frequently happens that this quantity cannot be expressed by an exact number of times the unit of measure. We must, therefore, adopt some simple method of expressing such quantities by means of figures.

For example, suppose that the unit of measure is a stick one yard long, and that we wish to ascertain the number of yards in a *piece of cloth*. Let us suppose that we can apply the unit

of measure 9 times to the cloth, and that there is a remainder less than 1 yard.

Now, we must form our idea of the magnitude of this remainder, by finding *what part* it is of the unit of measure. If the remainder is half as long as the unit of measure, we know its relative length. We will divide the unit of measure into eight equal parts; each of these parts is called an *eighth*. Let us suppose that we can apply one of these equal parts exactly 5 times to the remainder. The remainder, then, is equal to *five eighths* of the unit of measure. The whole length of the cloth is *nine yards and five eighths of a yard.*

This *five eighths* is called a *fraction*. A fraction, then, is a part of a unit. The figure which shows the number of equal parts into which the unit is divided is written under a horizontal line, and that which shows how many of these parts are taken, is written above the horizontal line. Thus, *five eighths* is expressed $\frac{5}{8}$, *three fourths* is expressed $\frac{3}{4}$. *Three and two fifths* is expressed $3\frac{2}{5}$. The sign of addition is understood between 3 and $\frac{2}{5}$.

EXAMPLES IN NOTATION.

Express in figures the following numbers:

1. Four hundred and forty-three.
2. Five hundred and three.
3. Two thousand eight hundred and forty-five.
4. Nine thousand four hundred and two.
5. Seven thousand and sixty-three.
6. Eight thousand four hundred and one.
7. Twelve thousand five hundred and twenty-two.
8. Fourteen thousand and five.

9. Twenty-six thousand four hundred and one.
10. Three hundred and fifty-one thousand four hundred and two.
11. Seven hundred thousand and seven.
12. Eight millions and forty-five.
13. Forty-five millions two hundred and six.
14. Three hundred and six millions and twenty-five.
15. Nine thousand and one.
16. Forty-five million two hundred and sixty-four.
17. Five trillions two thousand and five.
18. Eight hundred thousand and one.
19. Nine hundred and seven millions and three.
20. Forty thousand two hundred and one.

ROMAN NOTATION.

3. The Romans, in their system of notation, employed letters to express numbers. They used the following letters; namely, I. for *one;* V. for *five;* X. for *ten;* L. for *fifty;* C. for a *hundred;* D. for *five hundred.;* M. for a *thousand.* The other numbers they expressed by various repetitions and combinations of these seven letters, in the following manner:

I.	=	1.	D. or IƆ	=	500
II.	=	2.	M. or CIƆ	=	1000
III.	=	3.	MM.	=	2000
IV.	=	4.	$\overline{\text{V}}$. or IƆƆ	=	5000
V.	=	5.	$\overline{\text{VI}}$.	=	6000
VI.	=	6.	$\overline{\text{X}}$.	=	10000
VII.	=	7.	IƆƆƆ or $\overline{\text{L}}$.	=	50000
VIII.	=	8.	$\overline{\text{LX}}$.	=	60000

IX. =	9.	CCCIƆƆƆ or $\overline{\text{C}}$. =	100000
X. =	10.	$\overline{\text{M}}$. =	1000000
L. =	50.	$\overline{\text{MM}}$. =	2000000
C. =	100.	&c.	&c.

By examining the above table, we may make the following observations:

1. When any character is repeated, its value is repeated; thus, I. = 1, and by repeating I. we have II. = 2. Hence, XX. = 20, XXX. = 30.

2. When a letter is placed before another of greater value, the value of the latter is diminished by the value of the former. Thus, IV. = 4.

3. When a letter is placed after another of greater value, the value of the latter is increased by the value of the former. Thus, VI. = 6.

4. Every Ɔ placed at the right of IƆ increases its value ten times. Thus, IƆƆƆ = 50000.

5. When C is placed at the left hand of CIƆ, and Ɔ is placed at the right hand of the same, its value is increased ten times. D is now used instead of IƆ, and M. is used instead of CIƆ.

6. A bar placed over any letter causes it to represent a value a thousand times as great. Thus, $\overline{\text{VII}}$. = 7000.

The Roman Notation is not used for the purpose of making the numerical calculations in arithmetic. It is principally used for denoting dates, numbering the chapters in books, marking the divisions on the faces of watches and clocks, and for designating the value of bank notes.

EXAMPLES IN ROMAN NOTATION.

Express by means of the Roman characters the following numbers:

1. Four hundred and eighty-five.
2. Two thousand eight hundred and forty-three.
3. Forty-five, eighty, one hundred and twenty-one.
4. Six thousand four hundred and twenty-one.
5. One thousand eight hundred and fifty-three.
6. Four hundred, forty-seven.
7. Thirty-one, forty-two, seventy-five, eighteen.
8. Two thousand and two.

NUMERATION.

4. By referring to the table in Article 1, the pupil will be able to read any proposed number, which is expressed by means of figures, without difficulty, even when it contains units of a higher order than trillions. Thus, the number which the figures in that table express, is read, *three trillions, two hundred and ten billions, nine hundred and eighty-seven millions, six hundred and fifty-four thousand, three hundred and twenty-one.*

In order to read any number with facility, we may commence at the right hand, and divide it into periods of three figures each, and then commence at the left hand and enunciate the name of each period. The first, or right-hand period, is called the *period of units*, the second, *the period of thousands*, the third, *the period of millions*, the fourth, *the period of billions*, the fifth, *the period of trillions*, the sixth, *the period of quadrillions*, the seventh, *the period of quintillions*, the eighth, *the period of sextillions.*

EXERCISES IN NUMERATION.

The pupil may read the following sentences:

1. The wire suspension bridge at Wheeling, Va., over the Ohio river, is 1380 feet long.

2. The wire suspension bridge at Nashville, erected across the Cumberland river, is 656 feet long, and the whole length of the bridge and embankment is 1956 feet. The elevation of the bridge above low water is 110 feet.

3. The wire suspension bridge over the Niagara river, $1\frac{1}{2}$ miles below the Falls, is 800 feet long, and 260 feet above the river.

4. In Ohio 1200 square miles are underlaid with iron, and it is estimated at 1,080,000,000 tons. The whole amount of iron manufactured in the States, during the year 1845, was estimated at 919,100 tons, the value of which was 41,734,610 dollars.

5. According to the census of 1850, there were, in the United States, 4,220,293 Methodists; 3,134,438 Baptists; 2,045,516 Presbyterians; 631,613 Episcopalians; 705,983 Roman Catholics; and 795,677 Congregationalists.

6. In 1850, the population of the city of New York was 515,507; Brooklyn, 96,838; Williamsburg, 30,780; Boston, 136,871; Philadelphia, 408,762; Baltimore, 169,054; Chicago, 29,963; Milwaukie, 20,061; Cincinnatti, O., 115,436; Rochester, 36,403; Albany, 50,763; Syracuse, 22,271.

7. During the year 1852, the expenditures of the government of the United States amounted to 46,007,893 dollars.

8. The number of pounds of tobacco raised in the United States, during the year 1850, was 199,739,746.

9. The amount of tea consumed in the United States in the year 1846, was 16,891,020 pounds.

10. The amount of coffee consumed in the United States in the year 1846, was 124,336,054 pounds.

11. Read the numbers in the following table:

1.	23456853	**6.**	245	**11.**	42045605
2.	4200605	**7.**	6802	**12**	3280021
3.	203805	**8.**	30605	**13.**	316784
4.	6304067	**9.**	580025	**14.**	4580231
5.	60458003	**10.**	6789705	**15.**	26825425

CHAPTER III.

ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION.

ADDITION.

5. Addition is the finding of a single number which shall contain the total number of units in several given numbers. The number sought, is called the *sum* or *amount*. It is obvious that we can only add numbers that are related to the *same unit of measure*. In order to show how several numbers may be added, we will propose the following problem:

A man has two farms, one of which contains 473 acres, and the other 875 acres, How many acres do both farms contain?

To obtain the required number of acres, we must add 473 to 875.

We first set the numbers down, so that the units of the same order may be in the same vertical column. Then we say that 5 units and 3 units are 8 units, and place the 8 under the column of units. Passing to the column of tens, we say that 7 tens and 7 tens are 14 tens, which are equal to 1 hundred and 4 tens. We place the 4 tens under the column of tens, and add the 1 hundred to the column of hundreds. Now 8 hundred and 1 hundred make 9 hundred, and 4 hundred make 13 hundred, which are equal to 1 thousand and 3 hundred. Place the 3 hundred under the

Operation.

```
  473
  875
 ----
 1348 acres.
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column of hundreds, and the 1 thousand at the left. Hence, the whole number of acres in both farms is 1348.

Hence, for adding numbers, we have the following

RULE.

Set the numbers down so that the units of the same order may be in the same vertical column, and draw a line under the lowest number. Then add the units in the first, or right hand column, and if the sum be less than ten, set it down under that column; but, if it be greater than nine, set down the right hand figure in the sum directly under the column of figures added, and add the other figure, or figures, in this sum to the next column. Proceed in the same manner with each of the successive columns, and set down the total amount of the last column.

EXAMPLES.

1. What is the sum of 253, 986, 785, 253, 987, 30602, 4684, and 3061? *Ans.* 41611.

2. What is the sum of 304, 588, 2756, 375, 825, 3712, 9826, and 203? *Ans.* 18589.

3. What is the sum of 253, 6701, 208, 265, 5820, 3845, 2508, 7968, and 345? *Ans.* 27913.

4. What is the sum of 4685, 3852, 3064, 2075, 1345, 8456, 8235, 2035, 2605, 2041, and 2052? *Ans.* 32445.

5. What is the sum of 5852, 6834, 2583, 60502, 30402, 203, 845, 3689, 785, 275, 321, and 2531? *Ans.* 114822.

6. What is the sum of 25684, 38452, 7841, 2364, 9872, 3002, 40602, 3450, and 4238? *Ans.* 135505.

7. What is the sum of 38457, 68457, 2506, 8459, 30458, 2031, 6082, 2034, 2056, and 101? Ans. 160641.

8. What is the sum of 2385, 1720, 45868, 2045, 485, 3856, 2585, 453, and 68521? *Ans.* 127918

9: What is the sum of 3856, 25868, 2034, 7825, 30856, 20458, 45835, 2812, 4582, 30123, and 6789? *Ans.* 181038.

10. What is the sum of 34506, 280, 3061, 460, 385, 698, 483, 285, 6854, 893, 8945, and 3021? *Ans.* 59871.

11. A merchant owes to A $1500; to B $408; to C $1310; to D $50; and to E $1900; what is the sum of all his debts? *Ans.* $5168.

12. A cashier receives six bags of money; the first held $1034; the second, $1025; the third, $2008; the fourth, $7013; the fifth, $5075; and the sixth, $89. How much was the whole sum? *Ans.* $16244.

13. A man purchased a house for $2385, paid $385 for having it repaired, and then sold it so as to gain $500. What did he sell it for? *Ans.* $3270.

14. What is the sum of the following numbers: three thousand four hundred and sixty-five, two thousand and fifty-four, nine hundred and six thousand two hundred and forty-seven? *Ans.* 911766.

15. What is the sum of the numbers, one hundred and sixty-seven thousand, three hundred and sixty-seven thousand, nine hundred and six thousand, two hundred and forty-seven thousand, ten thousand, seven hundred thousand, nine hundred and seventy-six thousand, one hundred and ninety-five thousand, ninety-seven thousand? *Ans.* 3665000.

16. Europe contains 2,688,000 square miles; Asia, 14,128,-000; Africa, 8,720,000; Australia, 2,208,000; North America, 5,472,000; South America, 5,136,000. What is the whole number of square miles in these grand divisions of the globe? *Ans.* 38,352,000.

17. The age of A is 21 years; C 24 years; B 45 years; and D is 15 years older than B. What is the sum of their ages? *Ans.* 150 years.

18. A linen draper sold 10 pieces of cloth; the first contained 34 yards; the second, third, fourth and fifth, each 36 yards; the sixth, seventh, and eighth, each 33 yards; and the ninth and tenth, each 35 yards. How many yards did he sell? *Ans.* 347.

19. The length of the Mississippi river is about 3200 miles; the Missouri, 3100 miles; the Ohio, 1033 miles. What is the sum of their lengths? *Ans.* 7333 miles.

20. The population of London in 1850 was 2,240,000; New-York, 515,507; Paris, 1,053,897; Peking, 2,000,000; Canton, 1,000,000; Rome, 180,000; Liverpool, 400,000; New-Orleans, 145,449; Boston, 138,788; Mexico, 200,000. What is the entire population of these cities? *Ans.* 7,873,641.

21. A merchant bought a quantity of silk worth 584 dollars, and a quantity of cloth for 785 dollars. In selling, he gained 145 dollars on the silk, and 175 dollars on the cloth; for what amount did he sell both? *Ans.* 1689 dollars.

22. A gentleman bequeathed to his widow 3800 dollars, to each of his three son 1785 dollars, and to each of his two daughters 2352 dollars. What was the amount of his bequests? *Ans.* 13859 dollars.

23. Three men enter into partnership. The first man puts in 3845 dollars, the second 2375 dollars, and the third puts in 585 dollars more than the sums put in by the other two. What sum did they all put in? *Ans.* 13025 dollars.

24. From the year 1835 to the year 1852 inclusive, the sum of 47643 dollars was appropriated to Academies from the Lit-

erature Fund, by the Regents of the University of New York, for the purchase of books and philosophical apparatus designed for the use of such Academies. An equal sum was raised by the Academies. What amount of money was expended during these years for the purchase of books and apparatus for the use of these Academies? *Ans.* 95286 dollars.

25. In the year 1849 the expenditures of the United States were 57631667 dollars; in 1850, 43002168 dollars; in 1851, 48005879 dollars. What were the expenditures of the United States during these three years? *Ans.* 148639714 dollars.

26. In the year 1829 the expenditures of the United States were 12651489 dollars: in 1830, 13220534 dollars; in 1831, 13863768 dollars. What were the expenditures of the United States during these three years? *Ans.* 39735791 dollars.

27. In the year 1849 the receipts into the National Treasury from Customs, and the sales of Public Lands, were 31074347 dollars; in 1850, 43375798 dollars; in 1851, 52312979 dollars. What were the total receipts into the Treasury during these three years? *Ans.* 126763124 dollars.

28. In the year 1837 the receipts into the National Treasury from Customs, and the sale of Public Lands, were 18032846 dollars; in 1838, 19372984 dollars; in 1839, 30399043 dollars. What were the total receipts into the Treasury during these three years? *Ans.* 67804873 dollars.

29. In the year 1851, the expenditures of the Post Office Department were 6278402 dollars; the amount paid for the compensation of Postmasters was 1781686 dollars; and the amount paid for the transportation of the mail was 3538064 dollars. What did the expenses of the mail amount to for that year? *Ans.* 11598152 dollars

30. In the year 1840, the expenditures of the Post Office Department were 4718236 dollars; the amount paid for the compensation of Postmasters was 1028925 dollars; and the amount paid for the transportation of the mails was 3296876 dollars. What did the expenses of the mail amount to for that year? *Ans.* 9044037 dollars.

31. In the year 1849, the value of the exports from the United States were 145755820 dollars; in 1850, 136946912 dollars; and in 1851, 218388011 dollars. What was the whole amount of exports during these three years?

Ans. 501090743 dollars.

32. In the year 1849, the value of the imports into the United States were 147857439 dollars; in 1850, 178138318 dollars; in 1851, 216224932 dollars. What was the whole amount of imports during these three years?

Ans. 542220689 dollars.

We shall here give some tables which contain columns of figures for the pupil to add. He should be able to add the columns with rapidity. In adding a column of figures, do not name the figures to be added, since that requires too much time. Thus, in adding 8, 7, 6, 9, 4, do not say 8 and 7 are 15 and 6 are 21, and 9 are 30, and 4 are 34; but say 8, 15, 21, 30, 34. By practice, the pupil will be able to add two columns of figures at the same time. Thus, in adding 23, 18, 45, 67, he can say 23, 41, 86, 153, and so on. The pupil should also be able to add a single column of figures by deciding what is the sum of *more than two* figures by one perception. Thus, if 7, 8, 5, are the first three figures in a column, it will save time to perceive *at once* that their sum is 20. It will require twice as much time to say 15, 20.

TABLE I.*

THIS TABLE SHOWS THE VALUE OF THE IMPORTS AND EXPORTS OF EACH STATE, DURING THE YEAR ENDING JUNE 30, 1851.

	Value of Exports.		Total Value of Exports.	Value of Imports.
	Domestic Produce.	Foreign Produce.		
Maine	$ 1517487	$ 33951	$ 1551438	$ 1176590
New Hampshire	4949		4949	58028
Vermont	761712	304	762016	691262
Massachusetts	9857537	2495145	12352682	32715327
Rhode Island	223404	14373	237777	310630
Connecticut	433894	184	434078	342994
New York	68104542	17902477	86007019	141546538
New Jersey	139		139	1111
Pennsylvania	5101969	254067	5356036	14168761
Delaware				
Maryland	5416798	218988	5635786	6650645
District of Columbia	72560		72560	80813
Virginia	3087444	2624	3090068	522933
North Carolina	426748	4347	431095	206931
South Carolina	15316578		15316578	2081312
Georgia	9158879	1110	9159989	721547
Florida	3939910	262	3940172	94997
Alabama	18528824		18528824	413446
Louisiana	53968013	445950	54413963	12528460
Mississippi				845
Tenessee				64761
Missouri				622089
Ohio	395125		395125	686331
Kentucky				213576
Michigan	183448	7978	191426	182146
Illinois	114336		114336	4657
Texas	75422		75422	94715
California				13513
Oregon				
TOTAL	$	$	$	$

The symbol $ is placed before a number expressing *dollars.*

* *This and the following* table are taken from the American Almanac.

TABLE II.

THIS TABLE GIVES THE REVENUE AND EXPENDITURE OF THE POST-OFFICE FROM JULY 1ST, 1836, TO JUNE 30TH, 1845.

Year ending 30th June.	Letter Postage.	Newspapers and Pamphlets.	Total Annual Receipts.	Total Annual Expenditures.
1837	$3674834	$425714	$4236779	$3544630
1838	3776125	458737	4238733	4430662
1839	3976446	508873	4484657	4636536
1840	4003776	535229	4543522	4718236
1841	3812739	566246	4407726	4499528
1842	3953315	572225	4546849	5674752
1843	3738307	543277	4296225	4374754
1844	3676162	549744	4237288	4296513
1845	3660231	608765	4289841	4320732
TOTAL .	$	$	$	$

SUBTRACTION.

6. The process of finding the number by which one number exceeds another, is called subtraction. The less number is called the *subtrahend*, and the greater, the *minuend*. In order to show how one number may be taken, or subtracted from a greater, we will propose the following problem :

A man has two farms, one of which contains 383 acres, and the other 565. By how many acres does the larger farm exceed the smaller?

In order to obtain the required excess, we must subtract 383 from 565. In the first place, we set the less number under the greater, so that the units of the same order may be directly under each other, and draw a line under the subtrahend. We then say, 3 units taken from 5 units leave 2 units, which we write directly below the 3 units. In passing to

$$\begin{array}{r} 565 \\ 383 \\ \hline 182 \end{array}$$

the columns of tens, we meet with a difficulty, since we cannot take 8 tens from 6 tens. To remove this difficulty, we will take 1 hundred from the 5 hundred, and add its equal, 10 tens, to 6 tens, and we have 16 tens. Now 8 tens taken from 16 tens, leave 8 tens, which we write under the column of tens. Passing to the next column, we must observe that we have taken 1 hundred from the 5 hundred; hence, we have to take 3 hundred from 4 hundred; the remainder is 1 hundred. Therefore the excess sought is 1 hundred 8 tens and 2 units, or 182 acres.

From the solution which we have given of this question, we may notice that the two numbers may be disposed in the following manner:

		Hunds.		Tens.		Units.		
565	=	4	+	16	+	5		
383	=	3	+	8	+	3		
By subtracting, we have		1	+	8	+	2	=	182.

From this, we can see that the larger number exceeds the smaller by 1 hundred, 8 tens, and 2 units, or 182.

As another example, let it be required to subtract 158429 from 300405.

			10		
2	9	9	3	9	10
3	0	0	4	0	5
1	5	8	4	2	9
1	4	1	9	7	6

We cannot take 9 units from 5 units; and as there are no tens in the minuend, it is necessary to take 1 hundred from the 4 hundred, and observe that it is equal to 10 tens, or 9 tens and 10 units. The remainder, 3 hundred, we will write over the 4 hundred, the 9 tens over 0, and the 10 units over 5. If we add 10 units to 5 units, we shall have 15 units, from which we may

subtract 9 units; the remainder is 6 units, which we write under the column of units. Passing to the column of tens, we say that two tens taken from 9 tens leave 7 tens, which is the next figure in the remainder, and we write it under the tens. Now, as we cannot subtract 4 hundred from 3 hundred, and as the two following figures in the minuend are zeros, we must take 1 hundred thousand from the 3 hundred thousand, and observe that it is equal to 9 ten thousand, 9 thousand, and 10 hundred. The remainder, 2 hundred thousand, we will write over the 3 hundred thousand, the 9 ten thousand over the column of ten thousands, the 9 thousand over the column of thousands, and the 10 hundred over the column of hundreds. If we add 10 hundred to 3 hundred, we shall have 13 hundred, from which we may subtract 4 hundred; the remainder is 9 hundred, which we write under the column of hundreds for the next figure of the required remainder. In the two following subtractions, each of the zeros having been replaced by a nine, we say that 8 thousand taken from 9 thousand, leave one thousand, and that 5 ten thousand taken from 9 ten thousand, leave 4 ten thousand. Passing to the last column, we say, that one hundred thousand taken from two hundred thousand, leaves 1 hundred thousand. Hence, the required remainder is 141976.

From the solution of which we have given of this question, we may observe that the two numbers may be disposed in the following manner:

		Hun. Thou.		Tens Thou.		Thou.		Hunds.		Tens.		Units.
300405	=	2.	+	9	+	9	+	13	+	9	+	15
158429	=	1	+	5	+	8	+	4	+	2	+	9
		1	+	4	+	1	+	9	+	7	+	6

From this, we can see that the larger number exceeds the

smaller by 1 hundred thousand, 4 ten thousand, 1 thousand, 9 hundred, 7 tens, and 6 units, or 141976.

7. It is obvious that if we add the same number to the minuend and subtrahend, the remainder will not be effected. Let us make use of this observation in the following example:

From 485 subtract 296.

$$\begin{array}{r} 485 \\ 296 \\ \hline 189 \end{array}$$

Since we cannot subtract 6 units from 5 units, we add 10 units to 5 units, and then say, 6 units taken from 15 units leave 9 units, which we write under the units. As we added 10 units to the minuend, we must add its equal, 1 ten, to the subtrahend. Now, 9 tens and 1 ten are 10 tens, and as we cannot subtract 10 tens from 8 tens, we will add 10 tens to 8 tens; the sum is 18 tens, from which we subtract 10 tens, and set the remainder, 8 tens, directly below the column of tens. Since, in the last operation, we added 10 tens to the minuend, we must add its equal, 1 hundred, to the subtrahend. Now, 2 hundred and 1 hundred are 3 hundred. 3 hundred taken from 4 hundred leave 1 hundred. Hence, the required remainder is 189. This method of operating is much the most simple and easy in practice.

From the explanations which we have given, we may derive the following

RULE.

Set the subtrahend under the minuend in such a manner that the units of the same order may be in the same vertical column. Commence at the right hand, and subtract each figure in the subtrahend from the corresponding figure in the minuend, and set the remainder directly under the figure subtracted. If any figure in the subtrahend be greater than the

corresponding figure in the minuend, add 10 *to the figure in the minuend, and from the sum subtract the figure in the subtrahend. Add* 1 *to the next figure in the subtrahend, and then proceed as before.*

PROOF OF SUBTRACTION.

Add the remainder to the subtrahend; if the sum is equal to the minuend, the work is correct.

EXAMPLES.

The pupil may perform the subtraction in each of the first twelve examples, and then prove his work to be correct.

(1.)	(2.)	(3.)	(4.)
2345	78567	7856	67823
1217	3454	4278	25946

(5.)	(6.)	(7.)	(8.)
67857	758123	34268	27458
9869	96814	17259	16764

(9.)	(10.)	(11.)	(12.)
78364	741206	46833	52346
27259	312453	28476	27418

13. Newspapers were first published in 1630; how many years have they been published? *Ans.*

14. Cotton was first planted in the United States in the year 1769; how many years have elapsed since that time? How long before the Independence of the U. S. was cotton first planted? *Ans.*

15. A man sold a farm for $2350, and received a payment of $875; how much was then due to him? *Ans.* $1475.

16. The population of St. Louis in 1820, was 4598, and in 1850, it was 77860. What was the increase of population from 1820 to 1850? *Ans.* 73262.

17. A farmer had two fields of oats, each of which produced 275 bushels; another had two fields of oats, each of which produced 172 bushels. How many more bushels of oats did one farmer have than the other? *Ans.* 206 bushels.

18. The number of volumes in the library of Harvard University, is 92000; the number in the library of Yale College is 51000; the number in the library of Brown University is 31000; by how many volumes does the library of Harvard University exceed the number in the other two?

Ans. 10000 volumes.

19. In the year 1851, Harvard University had 6342 Alumni, and Yale College had 6114 Alumni. How many more Alumni had Harvard University than Yale College?

Ans. 228.

20. In 1850 the number of slaves in the United States was 3204067, and in 1790 the number of slaves was 697897. What has been the increase in the number of slaves during this period? *Ans.* 2506170.

21. A farmer paid $2345 for one farm, and $3485 for another, and then sold both farms for $4875. How much did he lose? *Ans.* $955.

22. A has $375 more than B, and $485 less than C, who has $3475. How many dollars have A and B together?

Ans. $5605.

23. A person paid $3750 for one farm, and $3680 for another.

He then sold the first for \$475 more than he gave, and the second for \$345 less than he gave for it. How much did he receive for both. *Ans.* \$7560.

24. I owe A \$345, C \$675, and D \$216. If I have \$3861, how much shall I have left after paying my debts? *Ans.* \$2625.

25. In 1852 the debt of the State of New York was \$21690802, and that of Pennylvania was \$40114236. How much greater was the debt of Pennsylvania than that of New York? *Ans.* 18423434.

26. In 1850 the coinage of the Mint of the United States was \$33892301, and in 1851 the coinage of the same was \$63488524. How much greater was the coinage of 1851 than that of 1850? *Ans.* \$29596223.

27. If the minuend is 3856, and the subtrahend 587, what is the remainder? *Ans.* 3269.

28. In 1836 there were sold 20074870 acres of public land, and in 1850 1846847 acres. How many more acres of the public land were sold in 1836 than in 1850? *Ans.* 18228023.

29. In Ohio the number of scholars who attended the common schools for the year 1851 was 445997; in New-York the number who attended the common schools for the year ending July 1, 1851 was 800430. How much does the latter number exceed the former? *Ans.* 354433.

30. If I borrow of my friend \$3860, and afterwards pay him \$999, how much is still due to him? *Ans.* 2861.

31. The value of all books, printed in English, which were imported into the United States during the year ending June 30, 1851, was \$384583; and the value of books, printed in

MULTIPLICATION.

8. When two numbers are given, the process of finding a third number which is composed of as many times the first as there are units in the second, or, of as many equal parts of the first, as there are like parts of a unit in the second, is called *multiplication.*

The first of the given numbers is called the *multiplicand*, and the second is called the *multiplier*. The two taken together, are called the *factors*, and the number sought is called the *product.*

$$\begin{array}{r} 13 \\ 13 \\ 13 \\ 13 \\ 13 \\ \hline 65 \end{array}$$

When the two factors are whole numbers, it is plain that we may obtain their product by successive additions. Thus, let it be required to multiply 13 by 5. Since the product must be composed of five times 13, we must find the sum of 5 numbers, each of which is 13. We find that this sum is 65. Hence, $13 \times 5 = 65$. In a similar manner, we may find the product of any two whole numbers, but this manner of operating is too long and tedious when the multiplier is composed of several figures; we will therefore seek for a better method of finding the product of two numbers; and multiplication properly consists in this abbreviated method.

Before we can present this method, it will be necessary for the pupil to commit to memory all the products that can be formed from the nine figures 1, 2, 3, 4, 5, 6, 7, 8, 9, by taking two of them at a time. These products are given in the following

TABLE.

2× 1= 2	3× 1= 3	4× 1= 4	5× 1= 5	6× 1= 6
2× 2= 4	3× 2= 6	4× 2= 8	5× 2= 10	6× 2= 12
2× 3= 6	3× 3= 9	4× 3= 12	5× 3= 15	6× 3= 18
2× 4= 8	3× 4=12	4× 4= 16	5× 4= 20	6× 4= 24
2× 5=10	3× 5=15	4× 5= 20	5× 5= 25	6× 5= 30
2× 6=12	3× 6=18	4× 6= 24	5× 6= 30	6× 6= 36
2× 7=14	3× 7=21	4× 7= 28	5× 7= 35	6× 7= 42
2× 8=16	3× 8=24	4× 8= 32	5× 8= 40	6× 8= 48
2× 9=18	3× 9=27	4× 9= 36	5× 9= 45	6× 9— 54
2+10=20	3×10=30	4×10= 40	5×10= 50	6×10= 60
2×11=22	3×11=33	4×11= 44	5×11= 55	6×11= 66
2×12=24	3×12=36	4×12= 48	5×12= 60	6×12= 72
7× 1= 7	8× 1= 8	9× 1= 9	11× 1= 11	12× 1= 12
7× 2=14	8× 2=16	9× 2= 18	11× 2= 22	12× 2= 24
7× 3=21	8× 3=24	9× 3= 27	11× 3= 33	12× 3= 36
7× 4=28	8× 4=32	9× 4= 36	11× 4= 44	12× 4= 48
7× 5=35	8× 5=40	9× 5= 45	11× 5= 55	12× 5= 60
7× 6=42	8× 6=48	9× 6= 54	11× 6= 66	12× 6= 72
7× 7=49	8× 7=56	9× 7= 63	11× 7= 77	12× 7= 84
7× 8=56	8× 8=64	9× 8= 72	11× 8= 88	12× 8= 96
7× 9=63	8× 9=72	9× 9= 81	11× 9= 99	12× 9=108
7×10=70	8×10=80	9×10= 90	11×10=110	12×10=120
7×11=77	8×11=88	9×11= 99	11×11=121	12×11=132
7×12=84	8×12=96	9×12=108	11×12=132	12×12=144

9. We shall first consider the most simple case in multiplication, namely, when the multiplier is a single figure.

Let it be required to multiply 387 *by* 8.

It is evident that the product demanded is composed of the sum of the products obtained by the taking successively 8 times the units of the multiplicand, 8 times the tens, and 8 times the hundreds. To obtain the sum of these products, we write the multiplier under the multiplicand, draw a line under them, and then proceed, as follows: 8 times 7 units are 56 units, or 5 tens and 6 units. We write the 6 units in the place of units, and retain the 5 tens to add to the product of the tens by 8. Then

$$\begin{array}{r} 387 \\ 8 \\ \hline 3096 \end{array}$$

we say, 8 times 8 tens are 64 tens, and the 5 tens retained, added to 64 tens make 69 tens, or 6 hundred and 9 ten. The 1 ten we write in the place of tens, and retain the 6 hundred to add to the product of the hundreds by 8. Finally, we say that 8 times 3 hundred are 24 hundred, to which we add 6 hundred, and we have 30 hundred, or 3 thousand and 0 hundred. We write the 0 hundred in the place of hundreds, and the 3 thousand in the place of thousands. Hence, we have 3096 for the product demanded.

10. We will now consider the case in which the multiplier consists of several figures. In order to present this case, we will propose the following question:

What is the product of 245 *by* 85?

$$\begin{array}{r} 245 \\ 85 \\ \hline 1225 \\ 19600 \\ \hline 20825 \end{array}$$

We commence by writing the multiplier under the multiplicand in such a manner that units of the same order may be in the same column, and then draw a line under the multiplier. Since $85 = 80+5$, we must take the multiplicand 5 times, then 80 times, and add the two products together for the entire product. We can find the former of these products as in the last article: it is 1225. Since $80 = 8$ times 10, it is plain, that if we multiply 245 by 8, and then multiply this produce by 10, we shall have the product of 245 by 80. The product of 245 by 8, may be found as in the last article: it is 1960. To multiply this last product by 10, we have only to place a 0 at its right, since from the principle of Notation, each figure in the multiplicand is thus made to represent 10 times as many units as it before represented. Hence, we have for the product of 245 by 80, the number 19600. If we now add together the

two partial products, 1225 and 19600, we shall obtain for the product demanded, 20825.

In practice we need not place the ciphers to the right of the partial products which we obtain by multiplying the multiplicand by the *tens*, *hundreds*, &c. in the multiplier, if we observe to place the first figure of each partial product in the same vertical column with the figure in the multiplier, which is one of the two factors of that product. Thus, in the preceding example, we can omit the cipher under the 5, without affecting the result.

11. It frequently happens that several of the figures of the multiplier are ciphers. In this case we can multiply the multiplicand by each significant figure in the multiplier, and arrange the partial products according to the direction in the last paragraph. For example, let it be required to multiply 2345 by 2003.

In the first place we multiply the multiplicand by 3, and obtain for the product, 7035. Now as there are no tens nor hundreds in the multiplier, we multiply by the figure 2, which expresses *thousands*, and obtain for the product 4690; and as it is necessary to make this product express *thousands*, we must arrange it under the first product so that its first figure may be in the same vertical column with 2, the figure in the multiplier. By adding the two partial products, we obtain for the product demanded, 4697035.

$$\begin{array}{r} 2345 \\ 2003 \\ \hline 7035 \\ 4690 \\ \hline 4697035 \end{array}$$

12. If one or both of the factors in multiplication are terminated by ciphers, we can abridge the operation by finding the product of the significant figures, and then annexing to the right of this product as many ciphers as there are at the right

hand of both factors. For example, let it be required to find the product of 247000 and 2300.

```
  247000
   2300
 -------
  741
 494
 ---------
568100000
```

After we have found the product of 247 by 23, we annex to the right hand of this product 5 ciphers. For if we multiply 247000 by 23, it is plain that the product will express *thousands.* Hence to the product of 247 by 23, we must annex 3 ciphers in order to obtain the product of 247000 by 23. Since 2300 equals 100 times 23, we must annex to the last product two ciphers, for the product demanded. The same reasoning will apply to all similar cases.

```
 43
  3
---
129
  5
---
645
```

13. Whenever the multiplier can be resolved into two factors, we can obtain the product by multiplying the multiplicand by one of these factors, and this product by the other. Thus the product of 43 by 15 may be found by multiplying 43 by 3, one of the factors of 15, and the product 129, by the other factor 5. If the multiplier can be resolved into more than two factors, we can proceed in a similar manner.

14. From what has been said, we may deduce the following rule for the multiplication of one number by another.

RULE.

I. *Write the multiplier under the multiplicand in such a manner that the first significant figure in the multiplier may be directly under the first significant figure in the multiplicand.*

II. *Multiply the multiplicand by each figure in the multiplier, and place the right hand figure of each partial product under the corresponding figure in the multiplier. When-*

ever the product of a figure in the multiplicand and a figure in the multiplier exceeds nine, set down the right hand figure of this product, and add the remaining part of this product to the product obtained by the same figure in the multiplier and the next figure in the multiplicand.

III. *Add together these partial products, and annex as many ciphers to the right hand of this sum as there are in both factors at the right hand of the significant figures, and we shall have the product demanded.*

EXAMPLES.

The pupil may perform the multiplication in the first six examples, and prove his work to be correct by addition.

(1.)	(2.)	(3)
2343	584568	413025
2	5	3

(4.)	(5.)	(6.)
200405	3046705	580467
4	6	7

7. What is the cost of 225 acres of land at 15 dollars per acre?

ANALYSIS.

Since 1 acre costs 15 dollars, 225 acres will cost 225 times 15 dollars, or 3375 dollars.*

* NOTE.—From this analysis, we see that 225 is the *multiplier*. It is important to observe that the *multiplier* must always be regarded as an *abstract* number, and that the *multiplicand* is a *concrete* number of the same denomination as the required product. But in practice, it is much more convenient to take that factor for the multiplier which

8. What is the cost of 223 tons of hay at 13 dollars per ton? *Ans* 2899 dollars.

9. If a person travel 43 miles per day, how far can he travel in 63 days at this rate? *Ans.* 2709 miles.

10. A press of 8 impression cylinders will print 16000 impressions per hour; how many impressions can such a press print in 24 hours? *Ans.* 384000.

11. If a student spend 45 dollars per year for the purchase of books, how many dollars will he spend in 36 years?

Ans. 1620 dollars.

12. How many bushels of wheat can be produced from 245 acres, if each acre produces 25 bushels? *Ans* 6125 bushels.

13. There have been counted in a single poppy 32000 seeds; how many would be found in 297 poppies? *Ans* 9504000.

14. There have been found in a single codfish 9344000 eggs; how many would there be in 35 such?

Ans. 327040000.

15. If 12 men can mow 168 acres in 14 days, in how many days can 1 man mow the same quantity? *Ans.*

16. The estimated cost of the New York and Erie Railroad is 51786 dollars per mile, and the length of the road is 464 miles; what is the whole cost of the road?

Ans. 24028704 dollars.

17. The estimated cost of the Hudson River Railroad is 64622 dollars per mile, and the length of the road is 144 miles; what is the cost of the whole road?

Ans. 9305568 dollars.

---

has the less number of *significant* figures. This may always be done, since the product of two factors may be obtained by taking either of the factors as a multiplier. This will appear plain upon reflection.

18. If 87 men can do a certain piece of work in 123 days, in how many days can 1 man do the same work?

Ans. 10701 days.

19. What is the cost of a farm containing 257 acres, at 47 dollars per acre? *Ans.* 12079 dollars.

20. A person purchased a farm of 250 acres, at 45 dollars per acre. He afterwards sold 115 acres of it at 53 dollars per acre, and finally sold the remainder for 39 dollars per acre. Did he gain or lose by this transaction?

Ans. He gained 110 dollars.

21. What is the cost of 2385 tons of iron at 63 dollars per ton? *Ans.* 150255 dollars.

22. A merchant bought 23 pieces of broad cloth, each piece containing 48 yards, and for each yard he paid 5 dollars; what was the cost of the 23 pieces? *Ans.* 5520 dollars.

23. The Syracuse and Central Square Plank Road is 16 miles long, and it cost 1487 dollars per mile; what was the whole cost of the road? *Ans.* 23792 dollars.

24. The Rome and Oswego Plank Road cost 1300 dollars per mile, and it is 62 miles long; what was the whole cost of the road? *Ans.* 80600 dollars.

25. A cow costs 28 dollars, and a horse costs four times as much, lacking 19 dollars. What is the cost of both?

Ans. 121 dollars.

26. A farmer purchased a farm containing 425 acres, at 43 dollars per acre, and then sells 225 acres at 50 dollars per acre. For how much must he sell the remainder in order that he may make by the whole transaction 2345 dollars?

Ans. 9370 dollars.

27. It was found by experiment that a meadow which yielded

4480 lbs. per acre, was made to yield 5288 lbs. per acre, by dressing it with the sulphate of soda. How many more pounds would a meadow, containing 35 acres, yield when dressed with the sulphate of soda? *Ans.* 28280 pounds.

28. It has been found by experiment that a field of oats yielded 48 bushels per acre, and that when the same field was dressed with the nitrate of soda, at the rate of 112 lbs. per acre, the yield was 64 bushels per acre. How many more bushels of oats would a farmer obtain from a field of 65 acres, by dressing it with the nitrate of soda? *Ans.* 1040 bushels.

29. It has been found by experiment that a sheep which was fed in the open air, consumed 1912 pounds of turnips from Nov. 18 to March 9, and that a sheep of the same size, fed in an open shed, consumed, during the same time, 1394 pounds of turnips. How many pounds of turnips would a farmer save in a single winter, by feeding 345 sheep in an open shed?

Ans. 178710 pounds.

30. The velocity of sound is 1142 feet per second. Now if the interval between seeing a flash of lightning and hearing the thunder be 7 seconds, what is the distance of the cloud?

Ans. 7994 feet.

31. What is the cost of 2385 tons of Railroad iron at 57 dollars per ton? *Ans.* 135945 dollars.

32. The surface of a man of ordinary size is 2500 square inches, and the pressure of the atmosphere on each square inch is about 15 pounds. What is the whole pressure on the man?

Ans. 37500 pounds.

33. It is estimated that the average number of pores in a square inch of skin is 2800. What is the whole number of pores in the skin of a man of ordinary size? *Ans.* 7000000.

34. If a young man's salary is 600 dollars per year, of which he takes 45 dollars to purchase books, and 300 dollars for board and other expenses; how much money will he have at the end of 7 years? *Ans.* 1785 dollars.

35. At 55 dollars per ton, what will the rails for a Railroad cost, if the road is 75 miles long, and it takes 112 tons per mile? *Ans.* 462000 dollars.

36. A farmer purchased a farm of 325 acres, at 45 dollars per acre, and made a payment of 875 dollars. In order to make another payment, he sold 75 acres at 58 dollars per acre. How much did he owe on his farm after the second payment was made? *Ans.* 9400 dollars.

37. What will it cost to construct 246 miles of Railroad, at 25875 dollars per mile? *Ans.* 6365250 dollars.

38. A farmer exchanges with a merchant 25 bushels of clover seed at 9 dollars per bushel, for 54 yards of cloth at 4 dollars yer yard, and a certain amount of sugar; what did his sugar cost him? *Ans.* 9 dollars.

39. What is the cost of supporting 5250 soldiers for one year, if it cost 175 dollars to support one?
Ans. 918750 dollars.

40. If 75 oranges cost one dollar, how many can be bought for 325 dollars? *Ans.* 24375.

DIVISION.

15. When two numbers are given, the process of finding a third number, such, that if it be multiplied by one of the given numbers, the product obtained will be equal to the other, is

called *Division.* Division, therefore, is the reverse of multiplication. The object of division is, then, *to find one of two factors, when one of the factors and their product are given.*

As in the multiplication of whole numbers, the product is composed of as many times the multiplicand as there are units in the multiplier, we may infer that to divide one number by another, is *to seek how many times the first number, considered as a product, contains the second, considered as the multiplicand;* this number of times is always the multiplier.

The first definition of division belongs to all possible numbers, while the second belongs only to whole numbers. The denominations given to the terms in division have been drawn from the definition. Thus, the first number is called the *dividend,* since it is separated into equal parts; the second is called the *divisor;* the third is called the *quotient.** Sometimes the divisor is not contained an exact number of times in the dividend. The part of the dividend which remains is called the remainder.

It follows from the definitions of division, that the product of the divisor and quotient, added to the remainder, if any, is equal to the dividend. By this means we can test the accuracy of the division.

In multiplication the product may be regarded as the *dividend,* the multiplicand as the *divisor* or *quotient,* and the multiplier as the *quotient* or *divisor;* hence, for the proof in multiplication, we may divide the product by one of the factors, and if the operation is exact, the quotient will be the other factor.

16. We have seen that multiplication may be performed by

* From the Latin word *quotius.*

successive additions, and since division is the reverse of multiplication, it may be performed by *successive subtractions.* For example, let it be required to divide 60 by 12. As many times as 12 can be subtracted from 60, so many times 12 is contained in 60.

In this example we can make five subtractions, and therefore the quotient is 5. But this manner of operating is too long, especially if the dividend is much larger than the divisor. Let us therefore seek for an abbreviated method for dividing one number by another; and it is this abbreviated process that constitutes the *rule* of division.

60	
12	
48	1st remainder.
12	
36	2d remainder.
12	
24	3d remainder.
12	
12	4th remainder.
12	
0	5th remainder.

17. By recollecting all the different products that can be formed by taking at a time any two of the numbers, 1, 2, 3, 4, 5, 6, 7, 8, 9, it is easy to determine the quotient of the division of a number consisting of *one or two figures* by a number represented by a *single figure*, when this quotient can be represented by a *single figure.* For example, the division of 63 by 7, gives 9 for the quotient, since 9 times 7 are 63. The division of 43 by 8 gives 5 for a quotient, and 3 for a

remainder, since 5 times 8 are 40, and the sum of 40 and 3, the remainder, is 43, the dividend.

18. We shall now consider the case in which the dividend is composed of several figures, the divisor being a single figure.

$$\begin{array}{r} 135 \\ 7 \\ \hline 945 \end{array} \qquad \begin{array}{r} 7)945 \\ \hline 135 \end{array}$$

If we multiply 135 by 7 we obtain 945 for the product. Hence, if we divide 945 by 7, we shall obtain 135 for the quotient. The product, 945, is the sum of the three partial products obtained by multiplying the three figures in the multiplicand by the multiplier. Hence, if we divide each of these partial products by 7, and find the sum of these partial quotients, this sum will express the quotient of 945 divided by 7. If we decompose 945 into the partial products, we shall have

$$\begin{array}{ccccccc} & & \text{Hundreds.} & & \text{Tens.} & & \text{Units.} \\ 945 & = & 7 & + & 21 & + & 35 \end{array}$$

Now 7 is contained in 7 hundreds 1 time, in 21 tens 3 times, in 35 units, 5 times. Therefore the quotient is 1 hundred, 3 tens, and 5 units, or 135. The operation may be expressed in the following manner:

$$\begin{array}{ccccccc} & & \text{Hundreds.} & & \text{Tens.} & & \text{Units.} \\ 7)945 & = & 7 & + & 21 & + & 35 \\ \hline 135 & = & 1 & + & 3 & + & 5 \end{array}$$

In practice, we say that 7 is contained in 9 hundred 1 time, and there remain 2 hundred, or 20 tens, which we add to the 4 tens, and we have 24 tens. Then we say, 7 is contained in 24 tens 3 times, and there remain 3 tens, or 30 units, which we add to the 5 units, and we have 35 units. Finally, we say

that 7 is contained in 35 units 5 times. It may be observed that the 24 tens may be obtained by simply placing 2, the first remainder, before the 4 tens. In a similar way the 35 units may be obtained.

This process of division is called *Short Division.* From the preceding explanation, we may derive the following rule for this kind of division:

RULE.

I. *Write the divisor to the left of the dividend, and separate them by a curved line, and draw a straight line under the dividend.*

II. *Take as many of the left hand figures of the dividend for a partial dividend as will contain the divisor once, and not more than nine times. Place the quotient directly under the right hand figure of this partial dividend. If there be any remainder, conceive it to be prefixed to the next figure in the dividend, thus forming a second partial dividend. But if there is not any remainder, the next figure in the dividend will be the second partial dividend, with which proceed as with the first.*

III. *Whenever the divisor is not contained in one of the partial dividends, put a cipher in the quotient, and place to the right of this partial dividend the next figure in the dividend, thus forming the next partial dividend.*

19. Whenever the divisor is not contained an exact number of times in the dividend, the remainder must be divided, since it is a part of the dividend. This division may be expressed by writing the divisor under the remainder, with a horizontal line between them. In order to make this matter more plain, we give the solution of the following question:

If 263 *dollars be equally divided among* 4 *men, how many dollars will each receive?*

Since 263 dollars are to be divided equally among 4 men, each man must have one *4th* part of 263 dollars, or as many times 1 dollar as 4 is contained times in 263. By dividing 263 by 4, we obtain 65 for the entire part of the quotient, and the remainder is 3. If the remainder had been 1 dollar, it is plain that each man should receive one 4th part of it, or $\frac{1}{4}$ of a dollar. But since the remainder is 3 dollars, the part of the remainder which each man must receive is three times one 4th of a dollar, or $\frac{3}{4}$ of a dollar. Hence, each man will receive $65\frac{3}{4}$ dollars.

By practice, the pupil will soon be able to employ Short Division in dividing one number by another, when the divisor contains two figures.

EXAMPLES.

The pupil may perform the division in the first twelve examples, and prove the work to be correct by multiplication

(1.)	(2.)	(3.)
5)234565	6)234564	8)230256
(4.)	(5.)	(6.)
3)5843268	2)40056430	5)7583645
(7.)	(8.)	(9.)
9)2346372	7)5046237	8)74568232
(10.)	(11.)	(12.)
11)25685121	12)645853644	13)2645539

13. How many barrels of flour, at 6 dollars a barrel, can be bought for 53418 dollars? *Ans.* 8903.

14. At the rate of 5 dollars per hat, how many hats can be bought for 625 dollars? *Ans.* 125 hats.

15. If a steam boat moves at the rate of 9 miles per hour, how many hours will it require to move over a distance of 234 miles? *Ans.* 26 hours.

16. At the rate of 9 dollars per yard, how many yards can be purchased for 23463 dollars? *Ans.* 2607 yards.

17. How many acres of land at 11 dollars per acre, may be bought for 2354 dollars? *Ans.* 214 acres.

18. There being 7 days in a week, how many weeks are there in 254 days? *Ans.* $36\frac{2}{7}$ days.

19. At 9 cents per bushel, how many bushels of sand may be bought for 5 dollars and 67 cents, or 567 cents?

Ans. 63 bushels.

20. A farmer has 4560 dollars, with which he wishes to purchase three farms for his three sons, Charles, Henry, and John. If the farms are of the same size, and he purchases the land at 5 dollars per acre, how many acres will each son have?

Ans. 304 acres.

21. If a man can travel 276 miles in 12 days, how many miles can he travel in one day? *Ans.* 23 miles.

22. If 25 laborers can pave a street in 24 days, how many days will 6 laborers require to pave the street?

Ans. 100 days.

23. There being 8 shillings in a dollar, how many bushels of wheat, at 7 shillings per bushel, can be bought for 2352 dollars? Ans. 2688 bushels.

24. A man spent 2400 dollars in buying oats at 3 shillings per bushel, and 1323 dollars in buying wheat at 9 shillings per bushel; how many more bushels of oats did he purchase than of wheat? *Ans.* 5224 bushels.

25. If 8 masons can build a wall in 75 days, in how many days can 3 masons build the same wall? *Ans.* 200 days.

26. If a barrel will contain 3 bushels, how many barrels will be required to contain 23472 bushels of apples?

Ans. 7824 barrels.

27. A man purchased two tracts of land. The whole cost of one of the tracts, at 11 dollars per acre, was 3421 dollars, and the whole cost of the other, at 13 dollars per acre, was 2717 dollars; what was the whole number of acres purchased?

Ans. 520 acres.

28. The expenditures of the United States in 1849 amounted to 57631667 dollars; in 1850, to 43002168 dollars; in 1851, to 48005879 dollars. How much did the expenditures average per year for these three years? *Ans.* 49546571$\frac{1}{3}$ dollars.

29. If 23813 dollars will employ one laborer for 23813 days, for how many days can 13 laborers be employed for the same sum? *Ans.* 1831$\frac{10}{13}$ days.

30. If a bushel of wheat is worth 125 cents, how many pounds of sugar, at 8 cents per pound, can be purchased for 15 bushels of wheat? *Ans.* 234$\frac{3}{8}$ pounds.

LONG DIVISION.

20. When the divisor is composed of several figures, it is not easy to make *mentally*, the several *multiplications* and *subtractions* that are necessary in the division of one number by

another. We therefore write down the different products and remainders. This mode of dividing one number by another is called *Long Division.* This method of division does not differ in principle from Short Division.

Let it be required to divide 5775 by 25.

OPERATION.

25)5775(231 = *the quotient.*
50
——
77
75
——
25
25
——
0 = *the remainder.*

EXPLANATION.

Since 25 is not contained in 5, the left hand figure of the dividend, it is necessary to take the two left hand figures of the dividend for a partial dividend. We find that 25 is contained in 57, 2 times, and the remainder is 7. Since 57 expresses *hundreds*, it is plain that 2, the first quotient figure, must express *hundreds.* To the remainder, 7 hundreds, we annex 7, the next figure in the dividend, and we have 77 for the next partial dividend. We find that 25 is contained in 77, 3 times, and the remainder is 2. Since 77 expresses *tens*, the second quotient figure must express *tens.* To the remainder, 2 tens, we annex 5, the next figure in the dividend, and we have 25 for the next partial dividend. The divisor is contained in this partial dividend *once,* and since 25 expresses units, the third

quotient figure must express *units*. Hence, the quotient demanded is 231.

Sometimes all the figures in the quotient are not significant figures. For example, let it be required to divide 87550 by 425. For this division we have the following

OPERATION.

```
425)87550(206
    850
    ---
     2550
     2550
     ----
```

We find that 425 is contained in 875 2 times, and the remainder is 25. To the remainder, 25, we annex 5 from the dividend, and obtain 255, for the next partial dividend. This dividend expresses *tens*, and since the divisor is not contained in it, there can be no *tens* in the quotient, and therefore we place a cipher at the right of the last quotient figure for the next figure in the quotient. To the last partial dividend, 255, we annex the last figure in the dividend, and obtain 2550 for the next partial dividend. We find that 425 is contained 6 times in 2550; hence, the quotient is 206.

If the right hand figures of the dividend and divisor are ciphers, we may cancel the *same number* of ciphers from each before commencing the division, since, by doing this, we make the divisor as much smaller as we do the dividend; therefore the quotient will not be effected.

From the preceding explanations, we may derive the following rule for dividing one number by another, when the divisor *is composed of* several figures.

RULE.

I. *Write the dividend to the left of the divisor, and draw a curved line between them, and also to the right of the dividend.*

II. *Take as many of the left hand figures of the dividend for a partial dividend, as there are figures in the divisor, and and if the partial dividend so obtained is less than the divisor, take from the left of the dividend one more than the number of figures in the divisor, for a partial dividend.*

III. *Find how many times the divisor is contained in the first partial dividend, and write the figure which expresses this number of times to the right of the dividend for the first quotient figure.*

IV. *Multiply the divisor by the first quotient figure, and subtract the product from the partial dividend. Annex to the right of this remainder the next figure in the dividend, for the second partial dividend. Proceed with the second partial dividend as with the first.*

V. *Continue the series of operations till the unit figure of the dividend has been annexed to the remainder last found, and at each operation write the quotient figure obtained to the right of the preceding one. If there be a final remainder, proceed as in Article* 19.

21. When the divisor can be resolved into factors, we may divide the dividend by one of these factors, and the quotient thus obtained by another of these factors, and so on, till we have used each of the factors as a divisor. The quotient last obtained will be the quotient required. For example, let us divide 48 by 6. We first divide 48 by 3, one of the factors of 6, and obtain 16 for the quotient. We then divide 16 by

2, which is the other factor of 6, and obtain 8 for the quotient. Now, 8 is the quotient required, since it is evident that the half of the third part of any number is equal to the sixth part of that number. We may give a similar explanation in any other case.

The only difficulty that can arise, when we adopt this mode of division, consists in finding the remainder. In order to discover some means of obtaining the remainder, let us propose the following question:

How many piles of apples, each containing 70, *can be formed out of* 473 *apples?*

Since each pile contains 70 apples, there will be as many piles as 70 is contained times in 473. We will find the required quotient by employing the factors of 70, which are 2, 5, and 7, since $2\times5\times7=70$. If we divide 473 by 2, as represented in the margin, we shall find that we can form out of 473 apples 236 piles, each containing 2 apples, and that we have 1 apple remaining. If we now divide 236 by 5, we shall find that we can form 47 piles, each of which contains 5 of the piles last formed, or 10 apples. We find that the remainder of this division is 1 of the piles first formed, or two apples. Finally, if we divide 47 by 7, we shall find that we can form out of the 47 piles 6 piles, each of which contains 7 of the 47 piles, or $7\times5=35$ of 236 piles, or $35\times2=70$ apples. The remainder of this division is 5, that is, it is 5 of the 47 piles, each of which, as we have seen, contains 5 of the piles first formed, or $5\times5=25$. If to

$$
\begin{array}{rl}
2)\underline{473} & \\
5)\underline{236} & ..\ 1=\text{1st rem.} \\
7)\underline{47} & ..\ 1=\text{2d rem.} \\
6 & ..\ 5=\text{3d rem.}
\end{array}
$$

25 we add the second remainder, which is one of the piles first formed, we shall have 26 of the piles first formed, or $26 \times 2 =$ 52 apples. Finally, by adding the first remainder, 1 apple, to 52 apples, we have 53 apples, which is the true remainder required. The operation of finding the true remainder may be expressed in this manner:

$$\text{True Remainder} = (5 \times 5 + 1) \times 2 + 1$$

Hence we have the following rule for finding the remainder.

RULE.

Multiply the last remainder by the last divisor but one, and to the product add the preceding remainder; multiply this sum by the next preceding divisor, and to the product add the next preceding remainder. Proceed in this manner till you have passed backward through all the divisors and remainders to the first, and the number last found will be the remainder required.

EXAMPLES.

The pupil may perform the division in the first twelve examples, and prove the work to be correct by multiplication.

(1.)	(2.)	(3.)
21)6783(	17)4675(	45)2025(
(4.)	(5.)	(6.)
45)90180(	17)6545(	23)51382(
(7.)	(8.)	(9.)
256)884736(	999)454545(	125(40000(
(10.)	(11.)	(12.)
35)268535(	245)2385678(	686)685467(

13. If 1225 dollars be equally divided among 25 men, how many dollars will each receive? *Ans.* 49 dollars.

14. How many barrels of molasses, at 14 dollars per barrel, can be bought for 2954 dollars? *Ans.* 211 barrels.

15. How many days can 21 horses subsist on an amount of food that will last 1 horse 3864 days? *Ans.* 184 days.

16. A farm containing 275 acres cost 6875 dollars. What did it cost per acre? *Ans.* 25 dollars.

17. If a man's salary is 600 dollars per year, and his yearly expenses 375 dollars, how many years will elapse before he is worth 7025 dollars, if he is worth at the present time 2300 dollars? *Ans.* 21 years.

18. A reservoir which holds 10035 gallons has three pipes. The first discharges into it 248 gallons in an hour, and the second 175 gallons in the same time; but the third discharges out of it 200 gallons in an hour. In what time can the reservoir be filled if the three pipes run together? *Ans.* 45 hours.

19. The Baltimore and Ohio Railroad is 298 miles long, and the cost of the road was 16000000 dollars; what was the average cost of the road per mile? *Ans.* $53691\frac{82}{298}$ dollars.

20. If a man purchase a farm, containing 250 acres, for 10750 dollars, and sells the same for 12750 dollars, how much does he gain per acre? *Ans.* 8 dollars.

21. A book-seller has 29250 cents, with which he wishes to purchase an equal number of Arithmetics and Algebras. If he pays 50 cents for an Arithmetic, and 75 cents for an Algebra, how many of each can he purchase? *Ans.* 234 of each.

22. Two locomotives start at the same time, one from Albany and the other from Buffalo, and move towards each other.

The one from Albany moves at the rate of 25 miles per hour, and the one from Buffalo at the rate of 20 miles per hour. If the distance from Albany to Buffalo is 335 miles, and the locomotive from Albany consumes 2 hours, and that from Buffalo 1 hour, in making stops, how many hours will elapse before they will meet? *Ans.* 9 hours.

23. The salary of the President of the United States is 25000 dollars per year. How much can he spend daily, and save out of his salary 3100 dollars at the end of the year?

Ans. 60 dollars.

24. A farmer hires a man and his boy to labor for him. He pays the man 75 cents a day, and the boy 20 cents a day. They work together for a certain number of days, and on settlement, it appears that the wages of both amount to 2280 cents. How many days did they work? *Ans.* 24 days.

25. If the remainder is 45, the quotient 44, and the dividend 2377, what is the divisor? *Ans.* 53.

26. How many bushels of corn at 52 cents a bushel must be exchanged for 324 bushels of oats at 39 cents per bushel?

Ans. 243 bushels.

27. A farmer sells a merchant 72 pounds of butter at 18 cents per pound, and receives in payment, 5 gallons of molasses at 42 cents per gallon, 20 pounds of sugar at 8 cents per pound, and the balance in cloth at 25 cents per yard. How many yards should the farmer receive? *Ans.* $37\frac{1}{25}$ yards.

28. The price of a pair of boots is 18 shillings, and the price of a pair of shoes, 14 shillings. A person wishes to expend 48 dollars in purchasing an equal number of each. How many of each can he purchase, there being 8 shillings in a dollar?

Ans. 12 of each.

29. The quotient is 385, the divisor 263, and the remainder 45; what is the dividend? *Ans.* 101300.

30. Two steam boats which run between New York and Albany, start at the same time, the one from Albany and the other from New York. The one that leaves New York runs in still water at the rate of 16 miles per hour, and it is retarded by the current 2 miles per hour. The one that leaves Albany runs at the rate of 15 miles per hour in still water, and it is accelerated by the current 1 mile per hour. How many hours will elapse before these boats will meet, if the distance from Albany to New York is 160 miles? *Ans.* $5\frac{10}{30}$ hours.

31. A person exchanges 84 books, at 6 shillings a volume, for 24 books of a different kind. If each of the 84 books cost 5 shillings, what did each of the 24 books cost?

Ans. $17\frac{12}{24}$ shillings.

32. How many cows that are worth 25 dollars a head should be exchanged for 775 sheep that are worth 2 dollars a head?

Ans. 62 cows.

33. According to the census of 1850, the number of pounds of tobacco produced in the United States was 199739746. If the value of a pound is 35 cents, how many school houses at 750 dollars for each house, could be built with the proceeds of this amount of tobacco?

Ans. 93211 school houses. The remainder is 66110 cents.

34. A farmer purchased a farm for 12250 dollars, and sold it for 15925 dollars. By this transaction he gained 15 dollars per acre. How many acres did the farm contain, and at what price per acre did he sell his farm?

Ans. 245 acres, and the price, 65 dollars per acre.

35. In 1847 there were manufactured in Pennsylvania

380350 tons of iron. If 1 mile of railroad require 115 tons of iron, how many miles could be constructed with the amount of iron manufactured in Pennsylvania in 1847?

Ans. $3385\frac{75}{115}$ miles.

36. How many bushels of apples, at 27 cents per bushel, should be given for 57 pounds of tea at 63 cents per pound?

Ans. 133 bushels.

For additional exercises in division, the pupil may complete the following tables:

TABLE I.

This table shows the number of operatives employed by the principal cotton manufacturing establishments in the United States, together with the annual amount of wages paid the same from 1839 *to* 1848, *inclusive.*

Years.	Males.	Females.	Wages of Females.	Wages of Males.	Y'rly sal'y of females.	Y'ly s'ly of Males.
1839	15000	50000	9880000	4680000		
1840	15000	52000	10275000	4836000		
1841	13800	46000	9089600	4305000		
1842	16500	55000	10868000	5148000		
1843	17000	59000	11658400	4304000		
1844	20000	66000	13041600	6240000		
1845	22000	72000	11227200	6864000		
1846	23000	75000	14820000	7176000		
1847	25000	85000	16796000	7800000		
1848	27000	95000	18772000	8424000		

From what is given in the above table, find the yearly wages of a male operative, and those of a female operative, for each of the 10 years.

TABLE II.

THIS TABLE SHOWS THE NUMBER AND TONNAGE OF THE VESSELS BUILT IN EACH STATE AND TERRITORY OF THE UNITED STATES, FOR THE YEAR ENDING ON THE 30TH OF JUNE, 1850.

STATE.	Vessels Built.	Total Tonnage.	Average Tonnage of each Vessel.
Maine	326	91212	
Vermont	1	77	
Massachusetts . .	121	35836	
Rhode Island . . .	14	3587	
Connecticut . . .	47	4819	
New York . . .	224	58343	
New Jersey . . .	57	6202	
Pennsylvania . .	185	21410	
Delaware	16	1849	
Maryland . . .	150	15065	
District of Columbia	8	288	
Virginia	34	3584	
North Carolina . .	33	2652	
Georgia . . .	5	684	
Florida	2	80	
Alabama	3	114	
Louisiana	24	1592	
Kentucky	34	6461	
Missouri	5	1354	
Illinois	13	1691	
Ohio	31	5215	
Michigan	14	2062	
Texas	1	106	
Oregon	2	122	
New Hampshire .	10	6914	
Total			

CHAPTER IV.

PRIME NUMBERS, COMPOSITE NUMBERS, GREATEST COMMON MEASURE, LEAST COMMON MULTIPLE, CANCELLATION.

DEFINITIONS.

1. A *Prime Number* is a number which is not exactly divisible by any number except itself and unity; thus, 7, 13, and 17 are prime numbers.

2. A *Composite Number* is one which has one, or more than one divisor, beside itself and unity; thus, 24, 12, and 6 are composite numbers.

3. A *Common Divisor*, or *Measure*, of two or more numbers, is any number in which is contained an exact number of times in these numbers; thus, 2 is a common divisor of 6, 12, and 18.

4. The *Greatest Common Measure* of two or more numbers, is the greatest number which is contained an exact number of times in each of the numbers; thus, 7 is the greatest common measure of 21, 42, and 70.

5. A *Multiple* of any number is one which is exactly divisible by the given number; thus, 15 is a multiple of 5.

6. The *Least Common Multiple* of two or more numbers, is the least number which is exactly divisible by each of them; thus, 24 is the least common multiple of 3, 12, and 8.

7. One number is *prime to another*, when they have no common measure except unity; thus, 15 and 16 are prime to each other, although neither is a prime number.

8. *Cancellation* is the rejecting of common factors in the dividend and divisor.

PROBLEM I.

22. *To resolve any composite number into its prime factors.*

For the solution of this problem, it is plain that we have the following:

RULE.

Divide the given number by any prime number of which the given number is a multiple; then divide the quotient thus obtained by any prime number of which it is a multiple. Proceed in this manner till a quotient is obtained which is a prime number. The several divisors and the last quotient will be the prime factors required.

In the application of this rule, the pupil may find it convenient to consult the following table, which contains all the prime numbers that are not greater than 659. It might be indefinitely extended.

TABLE.

1	31	79	137	193	257	317	389	457	523	601
2	37	83	139	197	263	331	397	461	541	607
3	41	89	149	199	269	337	401	463	547	613
5	43	97	151	211	271	347	409	467	557	617
7	47	101	157	223	277	349	419	479	563	619
11	53	103	163	227	281	353	421	487	569	631
13	59	107	167	229	283	359	431	491	571	641
17	61	109	173	233	293	367	433	499	577	643
19	67	113	179	239	307	373	439	503	587	647
23	71	127	181	241	311	379	443	509	593	653
29	73	131	191	251	313	383	449	521	599	659

EXAMPLES.

1. What are the prime factors of 13860?

OPERATION.

```
2)13860
  2)6930
  3)3465
 11)1155
    5)105
     3)21
        7
```

Hence, the prime factors are 2, 2, 3, 3, 5, and 11.

2. What are the prime factors of 480?
Ans. 2, 2, 2, 2, 2, 3, and 5.

3. What are the prime factors of 360?
Ans. 2, 2, 2, 3, 3, and 5.

4. What are the prime factors of 540?
Ans. 2, 2, 3, 3, 3, and 5.

5. What are the prime factors of 860?
Ans. 2, 2, 5, and 43.

6. What are the prime factors of 448?
Ans. 2, 2, 2, 2, 2, 2, and 7.

7. What are the prime factors of 2310?
Ans. 2, 3, 5, 7, and 11.

8. What are the prime factors of 455? *Ans.* 5, 7, and 13

9. What are the prime factors of 1155? *Ans.*

10. What are the prime factors of 434? *Ans.*

PROBLEM II.

23. *To find the greatest common divisor of two or more numbers.*

It is obvious that the greatest common measure of two or more numbers is the product of all the factors which are common to the given numbers. Hence, for the solution of this problem, we have the following

RULE.

Resolve each of the given numbers into its prime factors. The product of all the factors which are common to the numbers, is the greatest common measure of these numbers.

When the prime factors of a number are large, it is not easy to discover these factors. We shall therefore give another solution to this problem. Before we can give this solution it will be necessary to state the following principles:

FIRST PRINCIPLE.

Any number which will exactly divide any other number, will also divide any multiple of this second number, without a remainder.

For example, 14 being divisible by 7, it is plain that if 5 times 14, or 70, be divided by 7, the quotient will be 5 times the quotient of 14 divided by 7.

SECOND PRINCIPLE.

If any number be decomposed into two parts, each of which is divisible by a second number, it will also be divisible by this second number.

For example, take the number 48, and decompose it into two parts, 30 and 18. Each of these parts is divisible by 6, the quotients being 5 and 3. As the quotient of 48 divided by 6 is evidently equal to the sum of these two partial quotients, 48 is divisible by 6.

THIRD PRINCIPLE.

If any number be decomposed into two parts, any number which will divide one of these parts and the given number, will also divide the other part.

For, the entire quotient being equal to the sum of the two partial quotients, if one of these quotients is *fractional*, the other being an *integer*, it would follow that an entire number is equal to a whole number increased by a fractional number, which is absurd.

Let it be required to find the greatest common measure of 425 *and* 323.

It is evident that the greatest common measure of these two numbers cannot be greater than 323, the smaller number. Hence, if 323 is an exact divisor of 425, it is the greatest common measure required. Let us therefore divide 425 by 323. We find that 323 is contained once in 425, and that the remainder is 102. Therefore 323 is not the greatest common measure of 323 and 425. We now say that *the greatest common measure of* 323 *and* 425 *is equal to the greatest common measure of the remainder,* 102, *and* 323.

```
323)425(1
    323
    102)323(3
        306
         17)102(6
            102
              0
```

For, since $425 = 323 + 102$, the greatest common measure

of 425 and 323 must be a common measure of 323 and 102. (Third Principle.) Hence, the greatest common measure of 323 and 425 cannot be greater than either of the numbers 323 and 102. By the second principle, any number that will divide 323 and 102 will also divide 425. Hence, the greatest common measure of 323 and 102 is equal to the greatest common measure of 425 and 323.

We will now find the greatest common measure of 102 and 323. From what has been shown, we know that if 102 is contained an exact number of times in 323, it is the greatest common measure of these two numbers, and consequently of 425 and 323. We find that 102 is contained 3 times in 323, and that the remainder is 17. Hence, 102 is not the greatest common measure of 323 and 102, or of 323 and 425.

From what has been shown, we know that the greatest common measure of 17 and 102 is the same as the greatest common measure of 102 and $323=3\times102+17$. Now the greatest common measure of 17 and 102 cannot be greater than 17; hence, if 17 is contained an exact number of times in 102, it is the greatest common measure of 102 and 17. We find that 17 is contained 6 times in 102. Hence, 17 is the greatest common measure of 17 and 102; therefore it is the greatest common measure of 102 and 323, of 323 and 425.

From the preceding investigation we derive the following rule for finding the greatest common measure of two numbers:

RULE.

Divide the greater number by the less, and then divide the divisor by the remainder, and continue to divide the last divisor by the last remainder till a quotient is obtained which is an entire number.

24. If the greatest common measure of three numbers be required, find the greatest common measure of two of them, then that of this common measure, and the other number will be the one demanded.

For, let us take the numbers 1998, 918, and 522. The greatest common measure of 1998 and 918 is 54, and the greatest common measure of 54 and 522 is 18. We say that 18 is the greatest common measure of the three numbers. For, since 54 is the greatest common measure of 1998 and 918, it is the product of all the common factors of these numbers.* For the same reason, 18 is the product of all the common factors of 54 and 522. Hence, 18 is also the product of all the common factors of the three numbers 1998, 918, and 522. Hence, it is their greatest common measure.

As we may extend this reasoning to more than two numbers, we deduce the following rule for finding the greatest common measure of more than two numbers.

RULE.

Find the greatest common measure of two of the given numbers by the preceding rule, then that of the common measure so found, and another of the given numbers. Proceed in this manner till all of the given numbers have been used.

The greatest common measure last found will be the one required.

EXAMPLES.

1. What is the greatest common measure of 246 and 372? *Ans.* 6.

---

* NOTE.—The greatest common measure of two or more numbers is obviously equal to the product of all their common factors.

2. What is the greatest common measure of 9061 and 5525? *Ans.* 221.

3. What is the greatest common measure of 336, 720, and 1736? *Ans.* 8.

4. What is the greatest common measure of 1512, 3024, and 4608? *Ans.* 72.

5. What is the greatest common measure of 45, 75, and 125? *Ans.* 5.

6. What is the greatest common measure of 1081 and 1175? *Ans.* 47.

7. What is the greatest common measure of 2259, 2761, and 4267? *Ans.* 251.

8. A field is 1750 yards wide, and 1875 yards long. What is the length of the longest string that will exactly measure both the width and length of the field?

Ans. 125 yards.

9. There is a street 354 rods long, and the land on one side of this street is owned by three persons, A, B and C. A has 102 rods fronting the street, B 114 rods, and C 138 rods. They agree to divide their land into village lots, in such a manner, that the lots shall be of the greatest width that will allow each person to form an exact number of lots out of his land. What is this width? *Ans.* 6 rods.

PROBLEM III.

25. *To find the least common multiple of two or more numbers.*

Let us take the numbers 12, 15, 36, and 8. If we resolve these numbers into their prime factors, we shall have

$$
\begin{aligned}
12&=2\times2\times3,\\
15&=3\times5,\\
36&=2\times2\times3\times3,\\
\text{and}\quad 8&=2\times2\times2.
\end{aligned}
$$

It is obvious that the least common multiple of these four numbers must contain, as factors, all their *different* prime factors, and that each of these prime factors must be taken as a factor *as many times* in the least common multiple, as it is found as such in that one of the four numbers which contains this factor the greatest number of times. Hence, the least common multiple of 12, 15, 36 and 8, is

$$2\times2\times2\times3\times3\times5=360.$$

We have, then, for the solution of this problem, the following

RULE.

Resolve each of the numbers into its prime factors. Select each of the different prime factors as many times as it is found as a factor in that one of the numbers which contains this factor the greatest number of times. The product of the factors so selected will be the least common multiple required.

26. In finding the several factors of the least common multiple of two or more numbers, the student will often find it convenient to make use of the following

RULE.

Arrange the numbers in a horizontal line, and divide two or more of them by any prime number greater than unity, which will measure them. Set the quotients and the undivided terms directly below in a horizontal line. Divide two or more of the numbers in the second horizontal line by any prime number greater than unity, and set the quotients and undivided numbers directly below in a horizontal line. Proceed in this manner till the numbers in the last horizontal line are prime to each other. The product of the numbers in this line and the divisors, is the least common multiple required.

To show the application of this rule, let us find the least common multiple of the numbers 12, 15, 36, 8. For, finding this multiple, we have the following

OPERATION.

3)12	15	36	8
2)4	5	12	8
2)2	5	6	4
1	5	3	2

Therefore, $3\times2\times2\times5\times3\times2=360$, the least common multiple.

27. The least common multiple of two numbers which are prime to each other, is evidently equal to their product. Thus, the least common multiple of 15 and 28 is equal to $15\times28=420$. For, if we resolve these two numbers into their prime factors, there can be no common factors to the two numbers, since they are prime to each other. Hence, by Art. 25, the

product of all the factors, or the product of the two numbers, is their least common multiple.

If we multiply one of two numbers, which are prime to each other, by a factor of the other number, it is plain that the least common multiple of the two numbers is the same as the least common multiple of this product and the other number. But this product and the other number have a common measure; hence, for finding the least common multiple of two numbers, we have the following

RULE.

Find the greatest common measure of the two numbers, and divide their product by this common measure; the quotient will be the least common multiple required.

To show the application of this rule, let it be required to find the least common multiple of 425 and 544. For finding the greatest common measure, we have the following

OPERATION.

```
425)544(1
    425
    ---
    119)425(3
        357
        ---
         68)119(1
             68
             --
             51)68(1
                51
                --
                17)51(3
                   51
                   --
                    0
```

Whence, $(425\times544)\div17=13600$ is the least common multiple required.

28. For finding the least common multiple of three or more numbers, we have the following

RULE.

Find the least common multiple of two of the numbers, according to the last rule, and then that of this multiple and another of the numbers. Proceed in this manner till each of the numbers has been used. The least common multiple last found, will be the one required.

Let the student make himself familiar with the different rules for finding the least common multiple.

EXAMPLES.

1. What is the least common multiple of 15, 24, and 36 ? *Ans.* 360.

2. What is the least common multiple of 16, 18, 24, and 80 ? *Ans.* 720.

3. What is the least common multiple of 28, 42, 84, and 56 ? *Ans.* 168.

4. What is the least common multiple of 25, 35, and 45 ? *Ans.* 1575.

5. What is the least common multiple of 13, 39, and 9 ? *Ans.* 117.

6. What is the least common multiple of 49, 14, and 84 ? *Ans.* 1176.

7. What is the least common multiple of 176 and 245 ? *Ans.* 43120.

8. **What is the least common multiple of 3003 and 4851?**
Ans. 63063.

9. **What is the least common multiple of 98, 105, and 112?**
Ans. 5880.

10. **What is the least common multiple of 187, 275, and 357?**
Ans. 98175.

CANCELLATION.

29. It is plain that if we divide the dividend and divisor by the same number, the value of the quotient will not be changed. Thus, if 48 is the dividend, and 24 the divisor, we can divide the dividend and divisor by the common factor 8, without changing the value of the quotient. The process of rejecting the common factors of the divisor and dividend, is called *Cancellation.* Whenever we can discover the common factors of the divisor and dividend by *inspection*, it is a great saving of labor to cancel them before performing the division. The student cannot apply cancellation to advantage till he has quickened his perceptions by much practice.

EXAMPLES.

1. What is the quotient of 13125 divided by 375?

For the sake of convenience we write the divisor under the dividend and draw a line between them. Then by resolving each into factors, we have

$$\frac{13125}{375}=\frac{5\times2625}{5\times\ 75}=\frac{5\times5\times525}{5\times5\times\ 15}=\frac{5\times5\times5\times105}{5\times5\times5\times\ 3}=$$

$$\frac{\cancel{5}\times\cancel{5}\times\cancel{5}\times\cancel{3}\times35}{\cancel{5}\times\cancel{5}\times\cancel{5}\times\cancel{3}}=35,\text{ the quotient required.}$$

We cancelled the common factors in the dividend and divisor by drawing a line across them. By practice the student will be enabled to cancel large factors in the dividend and divisor, and thus abridge the operation. In this example, the expert Arithmetician would see, at a glance, that 25 is a common factor of the dividend and divisor, and that it is contained 15 times in the divisor, and 525 times in the dividend.

2. What is the quotient of 4935 divided by 105?

Ans. 47.

3. What is the quotient of 13545 divided by 387?

Ans. 35.

4. What is the quotient of 295482 divided by 7986?

Ans. 37.

5. What is the quotient of $84\times64\times48$ divided by $36\times8\times4$? *Ans.* 224.

6. What is the quotient of $35\times75\times150\times16$ divided by $14\times25\times4$? *Ans.* 4500.

7. What is the quotient of $48\times81\times36$ divided by $72\times18\times3$? *Ans.* 36.

8. What is the quotient of $51\times68\times96$ divided by $17\times34\times24$? *Ans.* 24.

9. How many acres of land, at 32 dollars per acre, should be exchanged for 128 acres at 18 dollars per acre?

Ans. 72 acres.

10. How many bushels of oats, at 35 cents per bushel, should be exchanged for 560 bushels of wheat, at 85 cents per bushel? *Ans.* 1360 bushels.

11. How many yards of cloth, at 75 cents per yard, must be given for 325 yards, at 18 cents per yard.

Ans. 78 yards.

12. A farmer sells to a merchant 275 pounds of butter, at 15 cents per pound, for an equal number of yards of cloth of two different kinds. If one kind of cloth is worth 25 cents per yard, and the other, 20 cents per yard, how many yards of each kind should the farmer receive for his butter?

Ans. $91\frac{2}{3}$ yards.

13. A farmer exchanged 175 bushels of wheat at 78 cents per bushel, for two kinds of cloth. The first kind of cloth was worth 40 cents per yard, and the second kind was worth 25 cents per yard. He was to have twice as many yards of the first kind as of the second; how many yards of each did he receive?

Ans. 260 yards of the first kind, and 130 yards of second kind.

14. A bookseller exchanges 96 copies of Smith's Mechanics, at 80 cents per copy, for an equal number of copies of each of the following works: Life of the Duke of Wellington, at 90 cents per copy, Watson's Mental Arithmetic, at 10 cents per copy, and Smith's Political Economy, at 60 cents per copy. How many copies of each did he receive? *Ans.* 48.

CHAPTER V.

FRACTIONS.

DEFINITIONS.

1. A *Fraction* is an expression which is used to represent any number of the *equal parts* into which the unit is divided. Thus, $\frac{2}{5}$, $\frac{3}{7}$, $\frac{4}{9}$, are fractions. See Art. 2.

2. The figure below the line is called the *denominator*, because the *name* of the parts depends on this figure. If this figure is 8, the parts are called *eighths*, and if it is nine, the parts are called *ninths*. The denominator shows the number of parts into which the unit has been divided.

3. The figure above the line is called the *numerator*, because it shows the *number* of the equal parts of unity that are taken to form the fraction.

4. The numerator and denominator of a fraction, taken together, are called the terms of the fraction.

5. A *Proper Fraction* is one whose numerator is *less* than its denominator; as $\frac{2}{5}$. It is obvious that a Proper Fraction is less than unity.

6. An *Improper Fraction* is one whose numerator equals or exceeds its denominator; as $\frac{3}{3}$, $\frac{5}{4}$. When the numerator is equal to the denominator, it is plain that the fraction is equal to unity; when it is greater, the fraction is greater than unity.

7. A *Simple Fraction* is one whose numerator and denominator are whole numbers; as $\frac{2}{6}$, $\frac{9}{5}$.

8. A *Compound Fraction* is a fraction of a fraction; as, $\frac{1}{2}$ of $\frac{2}{5}$.

9. A *Mixed Number* is one that is composed of a whole number and a fraction; as, $3\frac{2}{5}$, $4\frac{5}{8}$, &c.

10. A *Complex Fraction* is one whose numerator, or denominator, or both are fractions, or mixed numbers; as, $\frac{\frac{2}{5}}{\frac{3}{4}}$, $\frac{\frac{4}{7}}{4}$, $\frac{2\frac{1}{4}}{3\frac{1}{5}}$, $\frac{5\frac{1}{6}}{8}$, $\frac{7}{6\frac{2}{9}}$.

11. The fractions which we shall consider in this chapter are called *Common or Vulgar Fractions.*

30. From what has been said in Art. 19, page 49, it is evident that a fraction may be regarded as *the quotient of its numerator divided by its denominator.* Hence, 3 times the fourth part of unity, or three fourths, and the fourth part of 3, or 3 divided by 4, are identical expressions. The value of a fraction, then, may be regarded as being the quotient of its numerator divided by its denominator.

31. From the definitions which have been given of numerator and denominator, the following consequences evidently result:

1. *The value of a fraction is not changed by multiplying or dividing both the numerator and the denominator by the same number.*

2. *The value of a fraction is multiplied by any number, by multiplying its numerator, or dividing its denominator by that number.*

3. *The value of a fraction is divided by any number, by dividing its numerator or multiplying its denominator by that number.*

REDUCTION OF FRACTIONS.

The Reduction of Fractions consists in changing their form, without altering their value.

32. A fraction is reduced to its lowest or most simple terms, when its numerator and denominator are prime to each other. Hence, for reducing a fraction to its lowest terms, we have the following

RULE.

Divide both terms of the fraction by their greatest common measure, and the quotients will be the terms of the fraction required.

Whenever we can discover, by inspection, a common measure of the terms of a fraction, it is better to reduce the fraction to lower terms, by dividing the numerator and denominator by this common measure. The rule may then be applied to the resulting fraction.

EXAMPLES.

1. Reduce the fraction $\frac{48}{144}$ to its lowest terms.

$$4)\frac{48}{144} = \frac{12}{36} \qquad 12)\frac{12}{36} = \frac{1}{3}$$

Hence $\frac{48}{144} = \frac{1}{3}$, the fraction required.

2. Reduce the fraction [illegible] to its lowest terms.

In this example, the greatest common measure of 255 and 272 is 17. By dividing both terms of the fraction by 17, we find that

$$\frac{255}{272} = \frac{15}{16}.$$

3. Reduce the fraction $\frac{665}{760}$ to its lowest terms. *Ans.* $\frac{7}{8}$.

4. Reduce the fraction $\frac{195}{299}$ to its lowest terms. *Ans.* $\frac{15}{23}$.

5. Reduce the fraction $\frac{144}{240}$ to its lowest terms. *Ans.* $\frac{3}{5}$.

6. Reduce the fraction $\frac{352}{364}$ to its lowest terms. *Ans.*

7. Reduce the fraction $\frac{1344}{1536}$ to its lowest terms. *Ans.*

8. Reduce the fraction $\frac{904}{1017}$ to its lowest terms. *Ans.*

9. A farmer pays 38 dollars for a piece of land. If the land was purchased at the rate of 57 dollars per acre, what part of an acre did he purchase? *Ans.* $\frac{2}{3}$.

10. A and B purchased a store for 3760 dollars. A pays 2350 dollars, and B the remainder. What part of the store does each own? *Ans.* A $\frac{5}{8}$, B $\frac{3}{8}$.

11. If the length of a man's step is 3 feet, and he makes 30 steps each minute, what part of a mile can he travel in 20 minutes, there being 5280 feet in a mile? *Ans.* $\frac{15}{44}$.

12. A man, who has 2100 dollars, spends 600 dollars. What part of his money was left? *Ans.* $\frac{5}{7}$.

33. To reduce mixed numbers to improper fractions, we have the following

RULE.

Multiply the whole number by the denominator of the fraction, add the numerator to the product, and write the denominator under this sum for the fraction required.

EXAMPLES.

1. Reduce $4\frac{2}{5}$ to an improper fraction.

Since there are 5 fifths in 1, in 4 there are 4 times $\frac{5}{5}$, or $\frac{20}{5}$, and $\frac{2}{5}$ added to $\frac{20}{5}$ make $\frac{22}{5}$. Hence, $4\frac{2}{5}=\frac{22}{5}$.

2. Reduce $27\frac{3}{5}$ to an improper fraction. *Ans.* $\frac{138}{5}$.

3. Reduce $283\frac{4}{13}$ to an improper fraction. *Ans.* $\frac{3683}{13}$.

4. Reduce $25\frac{6}{17}$ to an improper fraction. *Ans.* $\frac{431}{17}$.

5. Reduce $223\frac{2}{14}$ to an improper fraction. *Ans.* $\frac{3124}{14}$.

6. Reduce $3405\frac{3}{17}$ to an improper fraction. *Ans.* $\frac{57338}{17}$.

7. Reduce $245\frac{8}{25}$ to an improper fraction. *Ans.* $\frac{6133}{25}$.

8. Reduce $34564\frac{2}{13}$ to an improper fraction. *Ans.* $\frac{449334}{13}$.

9. Reduce 45 to a fraction having 21 for its denominator. *Ans.* $\frac{945}{21}$.

10. Reduce 64 to a fraction having 25 for its denominator. *Ans.* $\frac{1600}{25}$.

11. Reduce $2384\frac{2}{17}$ to an improper fraction. *Ans.*

12. Reduce $257\frac{3}{11}$ to an improper fraction. *Ans.*

84. To reduce an improper fraction to a whole or mixed number, we have the following

RULE.

Divide the numerator by the denominator, and the quotient will be the whole or mixed number required.

EXAMPLES.

1. Reduce $\frac{25}{4}$ to a mixed number.

Since $\frac{4}{4}$ make a unit, there are as many units in 25 *fourths*, *as 4 is contained* times in 25. $25\div4=6\frac{1}{4}$. Hence, $\frac{25}{4}=6\frac{1}{4}$.

2. Reduce $\frac{25865}{18}$ to a mixed number. *Ans.*

3. Reduce $\frac{456492}{13}$ to a mixed number. *Ans.*

4. Reduce $\frac{256885}{14}$ to a mixed number. *Ans.*

5. Reduce $\frac{12223}{9}$ to a mixed number, *Ans.*

6. Reduce $\frac{2583465}{25}$ to a mixed number. *Ans.*

7. Reduce $\frac{2385564}{45}$ to a mixed number. *Ans.*

8. Reduce $\frac{23104556}{17}$ to a mixed number. *Ans.*

9. Reduce $\frac{345641}{15}$ to a mixed number. *Ans.* $23042\frac{11}{15}$.

10. Reduce $\frac{25004552}{27}$ to a mixed number. *Ans.*

35. To reduce fractions having different denominations to equivalent fractions having a common denominator, we have the following

RULE.

Multiply all the denominators together for a common denominator, and then multiply the numerator of each fraction by the product of all the denominators except its own, for a new numerator.

This rule is founded on the principle enunciated in Art. 31, namely, that the value of a fraction is not changed by multiplying both of its terms by the same number. In the application of this rule, it will be noticed that the terms of each fraction are multiplied by the product of all the denominators except its own. Hence the rule is correct.

EXAMPLES.

1. Reduce $\frac{2}{5}$, $\frac{3}{4}$, and $\frac{7}{20}$ to equivalent fractions having a common denominator.

Here, $5\times4\times20=400$, the common denominator. $2\times4\times20=160$, *the new* numerator of $\frac{2}{5}$.

$3 \times 5 \times 20 = 300$, the new numerator of $\frac{3}{4}$.

$7 \times 5 \times 4 = 140$, the new numerator of $\frac{7}{20}$.

Hence, the equivalent fractions are, $\frac{160}{400}$, $\frac{300}{400}$, and $\frac{140}{400}$. In this operation, it will be observed that the denominator of the first fraction was multiplied by 4 times 20, or 80, and that the numerator of this fraction was multiplied by the *same* number. Similar observations might be made in regard to the other two fractions.

2. Reduce $\frac{2}{7}$, $\frac{3}{8}$, and $\frac{2}{5}$ to equivalent fractions having a common denominator. *Ans.* $\frac{80}{280}$, $\frac{105}{280}$, $\frac{112}{280}$.

3. Reduce $\frac{5}{9}$, $\frac{2}{4}$, and $\frac{1}{10}$ to equivalent fractions having a common denominator. *Ans.* $\frac{200}{360}$, $\frac{180}{360}$, and $\frac{36}{360}$.

4. Reduce $\frac{5}{17}$ and $\frac{3}{25}$ to equivalent fractions having a common denominator. *Ans.* $\frac{125}{425}$ and $\frac{51}{425}$.

5. Reduce $\frac{5}{23}$ and $\frac{17}{12}$ to equivalent fractions having a common denominator. *Ans.* $\frac{60}{276}$ and $\frac{391}{276}$.

6. Reduce $\frac{2}{3}$, $\frac{2}{11}$, and $\frac{3}{14}$ to equivalent fractions having a common denominator. *Ans.* $\frac{308}{462}$, $\frac{84}{462}$, and $\frac{99}{462}$.

7. Reduce $\frac{2}{7}$, $\frac{3}{13}$, and $\frac{3}{9}$ to equivalent fractions having a common denominator. *Ans.* $\frac{234}{819}$, $\frac{189}{819}$, and $\frac{273}{819}$.

8. Reduce $\frac{5}{42}$, $\frac{3}{19}$, and $\frac{3}{20}$ to equivalent fractions having a common denominator. *Ans.*

9. Reduce $\frac{3}{21}$, $\frac{3}{5}$, and $\frac{17}{18}$ to equivalent fractions having a common denominator. *Ans.*

10. Reduce $\frac{2}{13}$, $\frac{5}{32}$, $\frac{1}{15}$, and $\frac{5}{12}$ to equivalent fractions having a common denominator.

Ans. $\frac{11520}{74880}$, $\frac{11700}{74880}$, $\frac{4992}{74880}$, and $\frac{31200}{74880}$.

36. To reduce fractions to equivalent fractions having the *least common denominator*, we have the following

RULE.

Reduce each of the fractions to its lowest terms, and then take the least common multiple of the denominators for the least common denominator required. Divide the least common denominator by the denominator of each of the fractions, and multiply the quotient by the numerator; the products will be the numerators of the fractions required.

This rule depends upon the same principle as the one in the preceding article.

EXAMPLES.

1. Reduce $\frac{8}{12}$, $\frac{5}{9}$, and $\frac{10}{16}$ to equivalent fractions having the least common denominator.

These fractions, when reduced to their lowest terms, become $\frac{2}{3}$, $\frac{5}{9}$, and $\frac{5}{8}$. The least common multiple of the denominators is 72. We must now reduce the fractions to equivalent fractions having 72 for their denominator. To do this, we must multiply both terms of each fraction by a number such that the denominator of the reduced fraction may be 72. It is plain that such a number may be found, for each fraction, by dividing the least common denominator by its denominator. The operation for finding the numerators may be represented as follows:

$\frac{\overset{24}{\cancel{72}}\times 2}{\cancel{3}}=48$, the numerator of the first fraction.

$\frac{\overset{8}{\cancel{72}}\times 5}{\cancel{9}}=40$, the " " second "

$\frac{\overset{9}{\cancel{72}}\times 5}{\cancel{8}}=45$, the " " third "

Hence, the equivalent fractions are $\frac{48}{72}$, $\frac{40}{72}$, and $\frac{45}{72}$.

2. Reduce $\frac{3}{5}$, $\frac{5}{12}$, and $\frac{7}{15}$ to equivalent fractions having the least common denominator. *Ans.* $\frac{36}{60}$, $\frac{25}{60}$, and $\frac{28}{60}$.

3. Reduce $\frac{5}{32}$, $\frac{8}{48}$, and $\frac{3}{72}$ to equivalent fractions having the least common denominator. *Ans.* $\frac{15}{96}$, $\frac{16}{96}$ and $\frac{4}{96}$.

4. Reduce $\frac{3}{17}$, $\frac{5}{15}$, and $\frac{4}{51}$ to equivalent fractions having the least common denominator. *Ans.* $\frac{135}{765}$, $\frac{255}{765}$, and $\frac{60}{765}$.

5. Reduce $\frac{5}{42}$, $\frac{3}{28}$, and $\frac{5}{14}$ to equivalent fractions having the least common denominator. *Ans.* $\frac{10}{84}$, $\frac{9}{84}$, and $\frac{30}{84}$.

6. Reduce $\frac{3}{13}$, $\frac{5}{26}$, and $\frac{7}{39}$ to equivalent fractions having the least common denominator. *Ans.* $\frac{18}{78}$, $\frac{15}{78}$, and $\frac{14}{78}$.

7. Reduce $\frac{15}{18}$, $\frac{2}{3}$, $\frac{7}{12}$, and $\frac{5}{36}$ to equivalent fractions having the least common denominator. *Ans.* $\frac{30}{36}$, $\frac{24}{36}$, $\frac{21}{36}$, and $\frac{5}{36}$.

8. Reduce $\frac{3}{25}$, $\frac{17}{45}$, and $\frac{19}{75}$ to equivalent fractions having the least common denominator. *Ans.* $\frac{27}{225}$, $\frac{85}{225}$, and $\frac{57}{225}$.

9. Reduce $\frac{2}{7}$, $\frac{3}{8}$, and $\frac{5}{56}$ to equivalent fractions having the least common denominator. *Ans.* $\frac{16}{56}$, $\frac{21}{56}$, and $\frac{5}{56}$.

10. Reduce $2\frac{3}{5}$, $1\frac{6}{7}$, and $7\frac{5}{15}$ to equivalent fractions having the least common denominator.* *Ans.* $\frac{273}{105}$, $\frac{195}{105}$, and $\frac{770}{105}$.

11. Reduce $\frac{21}{45}$ and $\frac{7}{90}$ to equivalent fractions having the least common denominator. *Ans.* $\frac{42}{90}$ and $\frac{7}{90}$.

12. Reduce $\frac{23}{56}$ and $\frac{41}{84}$ to equivalent fractions having the least common denominator. *Ans.* $\frac{69}{168}$ and $\frac{82}{168}$.

ADDITION OF FRACTIONS,

37. It is plain that fractions having a common denominator may be added by finding the sum of their numerators, and

* NOTE.—The mixed numbers must first be reduced to improper *fractions.*

placing this sum, as a numerator, over their common denominator. Thus it is as obvious that the sum of 3 *sevenths* and 2 *sevenths* is 5 *sevenths*, as it is that the sum of 3 and 2 is 5; that is, $\frac{3}{7}+\frac{2}{7}=\frac{5}{7}$. Hence, for the addition of fractions, we have the following

RULE.

*If the fractions have a common denominator, find the sum of their numerators, and place this sum, as a numerator, over the common denominator. If they have not a common denominator, reduce them to equivalent fractions having a common denominator, and then proceed as before.**

To add *mixed numbers*, we may reduce them to improper fractions, and then apply the rule; or, we may find the sum of the whole numbers and that of the fractions, and then add these two sums.

EXAMPLES.

1. What is the sum of $\frac{3}{5}$, $\frac{2}{7}$, and $\frac{4}{15}$?

By reducing the fractions to equivalent fractions having the least common denominator, we find that

$$\frac{3}{5}=\frac{63}{105},$$
$$\frac{2}{7}=\frac{30}{105},$$
$$\text{and } \frac{4}{15}=\frac{28}{105},$$

Hence, $\frac{3}{5}+\frac{2}{7}+\frac{4}{15}=\frac{63}{105}+\frac{30}{105}+\frac{28}{105}=\frac{121}{105}$.

* NOTE.—In the application of this rule the pupil should endeavor to discover, by inspection, the common denominator. Neither the slate nor the black-board should be used, when the calculation can be performed MENTALLY.

2. What is the sum of $13\frac{2}{21}$, $140\frac{3}{7}$, and $145\frac{5}{14}$.

Here $13+140+145=298$,

and $\frac{2}{21}+\frac{3}{7}+\frac{5}{14}=\frac{4}{42}+\frac{18}{42}+\frac{15}{42}=\frac{37}{42}$.

Hence the sum required is $298\frac{37}{42}$.

3. What is the sum of $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{7}{12}$? *Ans.* $\frac{47}{24}=1\frac{23}{24}$.

4. What is the sum of $\frac{3}{10}$, $\frac{2}{11}$, $\frac{3}{5}$, and $\frac{3}{4}$? *Ans.* $\frac{403}{220}=1\frac{183}{220}$.

5. What is the sum of $\frac{5}{17}$, $\frac{7}{51}$, and $\frac{9}{68}$? *Ans.* $\frac{115}{204}$.

6. What is the sum of $3\frac{2}{3}$, $17\frac{3}{4}$, and $28\frac{5}{12}$? *Ans.* $49\frac{5}{6}$.

7. What is the sum of $214\frac{1}{3}$, $517\frac{7}{48}$, $145\frac{5}{12}$? *Ans.* $876\frac{43}{48}$.

8. What is the sum of $\frac{23}{217}$ and $\frac{61}{98}$? *Ans.* $\frac{2213}{3038}$.

9. What is the sum of $245\frac{1}{3}$, $2896\frac{4}{9}$, and $49\frac{14}{27}$?

Ans. $3191\frac{8}{27}$.

10. What is the sum of $3\frac{5}{6}$, $17\frac{2}{7}$, and $4\frac{8}{21}$? *Ans.* $25\frac{1}{2}$

11. What is the sum of $\frac{3}{5}$, $\frac{5}{48}$, $\frac{7}{15}$, and $\frac{3}{16}$? *Ans.* $1\frac{43}{120}$.

12. A person bought 3 piles of wood; the first contained $45\frac{3}{8}$ cords, the second, $15\frac{3}{16}$ cords, and the third $25\frac{3}{4}$ cords. How many cords did he purchase? *Ans.* $86\frac{5}{16}$.

13. A man pays for a hat $4\frac{3}{4}$ dollars, for a coat $14\frac{1}{2}$ dollars, and for a pair of boots $4\frac{7}{8}$ dollars. How much money did he pay out? *Ans.* $24\frac{1}{8}$ dollars.

14. A merchant has 3 pieces of silk; the first contains $13\frac{3}{16}$ yards, the second, $21\frac{5}{8}$ yards, and the third $19\frac{3}{4}$ yards. How many yards are there in the three pieces? *Ans.* $54\frac{9}{16}$ yards.

15. A student paid $1\frac{1}{4}$ dollars for "Upham's Treatise on the Will," $3\frac{1}{2}$ dollars for "De Morgan's Differential and Integral Calculus," $2\frac{3}{4}$ dollars for "Fresenius' Qualitative Analysis," and $1\frac{5}{8}$ dollars for "Smith's Elementary Treatise on Mechanics." How much money did he pay out for his books?

Ans. $9\frac{1}{8}$ dollars.

16. A owns $\frac{3}{8}$ of a book-store, B, $\frac{2}{5}$, and C $\frac{1}{7}$. What part of the book-store do the three men own? *Ans.* $\frac{257}{280}$.

SUBTRACTION OF FRACTIONS.

38. To subtract one fraction from another, we have the following

RULE.

If the fractions have a common denominator, find the difference of their numerators, and place this difference as a numerator, over the common denominator. If they have not a common denominator, reduce them to equivalent fractions having a common denominator, and then proceed as before.

To subtract one mixed number from another, we may reduce the mixed numbers to improper fractions, and then apply the rule, or, subtract the fractional part of the subtrahend from the fractional part of the minuend, and to this difference, add the difference of the whole numbers.

EXAMPLES.

1. From $\frac{5}{7}$ take $\frac{2}{9}$.

Here $\frac{5}{7}=\frac{45}{63}$, and $\frac{2}{9}=\frac{14}{63}$; hence, $\frac{5}{7}-\frac{2}{9}=\frac{45}{63}-\frac{14}{63}=\frac{31}{63}$ the difference required.

2. From $14\frac{2}{5}$ take $3\frac{3}{7}$.

OPERATION.

$$\begin{array}{r} 14\frac{2}{5}=14\frac{14}{35} \\ 3\frac{3}{7}=\ 3\frac{15}{35} \\ \hline 10\frac{34}{35} \end{array}$$

In this example we reduce the fractional parts to equivalent

fractions having the same denominator, and find the fraction in the subtrahend to be larger than that in the minuend. We took 1 from 14, called it $\frac{35}{35}$, and added the $\frac{35}{35}$ to $\frac{14}{35}$, and obtained $\frac{49}{35}$. Now $\frac{15}{35}$ taken from $\frac{49}{35}$ leaves $\frac{34}{35}$, and 3 taken from 13 leaves 10. Hence, the remainder is $10\frac{34}{35}$.

3. From $\frac{5}{8}$ take $\frac{2}{5}$ *Ans.* $\frac{9}{40}$.

4. From $\frac{8}{15}$ take $\frac{5}{20}$. *Ans.*

5. From $\frac{7}{12}$ take $\frac{3}{24}$ *Ans.*

6. From $\frac{25}{43}$ take $\frac{21}{45}$. *Ans.*

7. From $\frac{6}{27}$ take $\frac{7}{36}$. *Ans.* $\frac{1}{36}$.

8. From $25\frac{2}{5}$ take $14\frac{1}{7}$. *Ans.* $11\frac{9}{35}$.

9. From $172\frac{5}{8}$ take $151\frac{7}{8}$. *Ans.*

10. From $256\frac{2}{5}$ take $28\frac{3}{7}$. *Ans.*

11. From 484 take $45\frac{17}{20}$. *Ans.*

12. From 325 take $14\frac{275}{380}$. *Ans.* $310\frac{105}{380}$.

13. A man bought a barrel of flour for $4\frac{3}{16}$ dollars, and sold it for $5\frac{7}{8}$ dollars. What did he gain by the transaction? *Ans.* $1\frac{11}{16}$.

14. A grocer bought a quantity of ham for $15\frac{3}{8}$ dollars, and a quantity of beef for $23\frac{3}{4}$ dollars. He sold the whole for $43\frac{7}{8}$ dollars. How much did he gain? *Ans.* $4\frac{3}{4}$ dollars.

15. Which is the greater $\frac{23}{85}$, or $\frac{31}{96}$. *Ans.* $\frac{31}{96}$.

16. A merchant has due to him 2380 dollars. He collects from A $275\frac{3}{8}$ dollars, from B $145\frac{1}{5}$ dollars, from C $320\frac{1}{4}$ dollars, and from D $500\frac{3}{10}$ dollars. How much is still due to him? *Ans.* $1138\frac{7}{8}$ dollars.

17. The length of a lot is $385\frac{5}{16}$ yards, and the width is $145\frac{7}{8}$ yards. What is the difference between the length and breadth? *Ans.* $239\frac{7}{16}$ yards.

18. From a hogshead of wine which contains 63 gallons, $33\frac{5}{16}$ gallons were drawn. How many gallons remained?

Ans. $29\frac{11}{16}$ gallons.

MULTIPLICATION OF FRACTIONS.

39. Let it be required to multiply $\frac{3}{5}$ by $\frac{7}{8}$.

We will first multiply $\frac{3}{5}$ by 7. 7 times $\frac{3}{5}$ are $\frac{21}{5}$. Since the multiplier 7 is 8 times as large as the given multiplier $\frac{7}{8}$, it follows that the product, $\frac{21}{5}$, is 8 times too large. Hence, we must divide $\frac{21}{5}$ by 8, in order to obtain the required product. Now a fraction may be divided by 8 by multiplying its denominator by 8. (Art. 31.) Hence, the required product is $\frac{21}{5\times 8}=\frac{21}{40}$. We observe that 21 is the product of the numerators 3 and 7, and that 40 is the product of the denominators 5 and 8. Hence, for the multiplication of fractions, we have the following

RULE.

Find the product of the numerators for the numerator, and the product of the denominators for the denominator of the required fraction.

This rule may be made to apply to every case in the multiplication of fractions by reducing mixed numbers to improper fractions, and regarding whole numbers as fractions having one for their denominator. If there are more than two fractions, we can apply the rule in finding their product, since we may regard the product of the first two as a single fraction by which we may multiply the third, and so on.

Since the value of a fraction is not changed by dividing

both of its terms by the same number, we may, in the application of this rule, *cancel* the factors which are common to the numerators and denominators.

EXAMPLES.

1. What is the product of $\frac{2}{3}$, $\frac{10}{12}$, $\frac{9}{15}$, and $\frac{16}{18}$?

OPERATION.

$$\frac{\cancel{2}}{3} \times \frac{\overset{\cancel{2}}{\cancel{10}}}{\underset{\cancel{4}}{\cancel{12}}} \times \frac{\overset{\cancel{3}}{\cancel{9}}}{\underset{\cancel{5}}{\cancel{15}}} \times \frac{\overset{8}{\cancel{16}}}{\underset{9}{\cancel{18}}} = \frac{8}{27}$$

2. What is the product of $2\frac{3}{5}$ and $3\frac{3}{4}$?

Since $2\frac{3}{5} = \frac{13}{5}$, and $3\frac{3}{4} = \frac{15}{4}$, $2\frac{3}{5} \times 3\frac{3}{4} = \frac{13}{\cancel{5}} \times \frac{\overset{3}{\cancel{15}}}{4} = 9\frac{3}{4}$.

3. What is the product of 48 and $7\frac{1}{8}$?

In this example we can multiply 48 by 7, and add $\frac{1}{8}$ of 48 to the product.

OPERATION.

$$\begin{array}{r} 48 \\ 7\frac{1}{8} \\ \hline 336 \\ 6 \\ \hline 342 \end{array} \quad \textit{Ans.}$$

4. What is the product of $\frac{3}{4}$, $\frac{2}{5}$, $\frac{15}{18}$, $\frac{7}{12}$, $\frac{8}{21}$, and $\frac{2}{5}$?

Ans. $\frac{1}{18}$.

5. What is the product of $\frac{3}{8}$, $\frac{2}{5}$, $\frac{17}{35}$, $\frac{19}{68}$, $\frac{7}{57}$, and $\frac{8}{14}$?

Ans. $\frac{1}{700}$.

6. What is the product of $\frac{13}{20}$, $\frac{4}{5}$, $\frac{8}{9}$, $\frac{6}{7}$, $\frac{35}{45}$, and $\frac{2}{5}$? *Ans.* $\frac{32}{225}$.

7. What is the product of $\frac{17}{40}$, $\frac{8}{9}$, $\frac{21}{12}$, $\frac{3}{7}$, and $\frac{19}{45}$? *Ans.* $\frac{323}{2700}$.

8. What is the product of 8, $2\frac{3}{5}$, $\frac{5}{39}$, $\frac{3}{10}$, and $2\frac{2}{3}$? *Ans.* $2\frac{2}{15}$.

9. What is the product of $18\frac{8}{9}$ and $2\frac{1}{2}$? *Ans.* $47\frac{2}{9}$.

10. What is the product of $8\frac{2}{3}$, $2\frac{3}{13}$, $3\frac{3}{4}$, and $16\frac{2}{3}$? *Ans.* $1208\frac{1}{3}$.

11. Reduce $\frac{2}{7}$ of $\frac{8}{9}$ of $\frac{12}{13}$ of $\frac{3}{2}$ to a simple fraction.* *Ans.* $\frac{32}{91}$.

12. Reduce $\frac{3}{8}$ of $\frac{15}{18}$ of $\frac{3}{5}$ of $\frac{2}{21}$ of $\frac{7}{9}$ to a simple fraction. *Ans.* $\frac{1}{72}$.

13. What is the cost of $5\frac{3}{8}$ yards of cloth at $3\frac{3}{4}$ dollars per yard? *Ans.* $20\frac{5}{32}$ dollars.

14. What is the cost of $17\frac{3}{5}$ tons of hay, at $9\frac{3}{4}$ dollars per ton? *Ans.* $171\frac{3}{5}$ dollars.

15. What will it cost to construct $64\frac{3}{8}$ miles of Railroad, at 30000 dollars per mile? *Ans.* 1931250 dollars.

16. What should I pay for $\frac{7}{8}$ of an acre of land, at $25\frac{3}{4}$ dollars per acre? *Ans.* $22\frac{17}{32}$ dollars.

17. What is the cost of $145\frac{1}{4}$ tons of coal, at $7\frac{7}{8}$ dollars per ton? *Ans.*

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* NOTE.—When two or more fractions are connected by the word *of*, the expression is called a *compound fraction*. The sign of multiplication may be substituted for the word *of*. For $\frac{2}{5}$ of $\frac{3}{7}$ is obviously equal to *two times one fifth of three sevenths*, that is $\frac{2}{5}$ of $\frac{3}{7}=2\times\frac{1}{5}$ of $\frac{3}{7}$. Now $\frac{1}{5}$ of $\frac{3}{7}=\frac{3}{7}\div 5=\frac{3}{5\times 7}$. (Art. 31.) Hence $\frac{2}{5}$ of $\frac{3}{7}=2\times\frac{3}{5\times 7}=\frac{2\times 3}{5\times 7}=\frac{6}{35}$. But $\frac{2}{5}\times\frac{3}{7}=\frac{2\times 3}{5\times 7}=\frac{6}{35}$. Therefore the sign $\times$ may be substituted for *the word of.*

18. What is the cost of 75 cords of wood, at $1\frac{7}{8}$ dollars per cord? *Ans.* $140\frac{5}{8}$ dollars.

19. Two travellers set out, at the same time, from the towns A and B, and travel towards each other till they meet, when it appears that each has travelled $8\frac{2}{3}$ hours. Allowing that one travelled $3\frac{1}{2}$, and the other $2\frac{3}{4}$ miles per hour, what is the distance from the town A to the town B? *Ans.* $54\frac{1}{6}$ miles.

20. A person purchases $87\frac{3}{10}$ acres of land at 45 dollars per acre, and $32\frac{7}{10}$ acres at 44 dollars per acre. What did he pay for his land?* *Ans.* $5367\frac{3}{10}$.

21. A man had a farm containing 344 acres. He sold $\frac{5}{8}$ of it at one time, and $117\frac{3}{17}$ acres at another time. How many acres did he have left? *Ans.* $11\frac{14}{17}$ acres.

22. A grocer bought 564 pounds of cheese at $6\frac{1}{4}$ cents per pound, and sold $\frac{1}{2}$ of it at $7\frac{3}{4}$ cents per pound, and the other half at $8\frac{3}{4}$ cents per pound. How much did he make on his cheese? *Ans.* 1128 cents.

23. A hatter purchased 24 hats, at $3\frac{7}{8}$ dollars per hat, and sold them at $4\frac{5}{8}$ per hat. How much was his gain?

Ans. 18 dollars.

24. What is the cost of $28\frac{11}{16}$ pounds of tea at 56 cents per pound? *Ans.* $1606\frac{1}{2}$ cents.

DIVISION OF FRACTIONS.

40. We have seen that a fraction may be divided by a *whole number*, by multiplying its denominator by the whole

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* *Note.*—This question will admit of a brief solution.

number. Let it now be required to *divide one fraction by another.* We will find the quotient of $\frac{5}{8}$ divided by $\frac{3}{7}$.

$$\tfrac{5}{8}\div 3=\tfrac{5}{8\times 3}=\tfrac{5}{24}.$$

But the divisor, 3, is *seven times as large* as the true divisor $\frac{3}{7}$; hence, the quotient, $\frac{5}{24}$, *is seven times too small*, and to obtain the true quotient, or the one demanded, we must, therefore, multiply $\frac{5}{24}$ by 7. Hence, the quotient required is

$$\tfrac{5\times 7}{24}=\tfrac{35}{24}.$$

Now it is easy to see that this quotient may be obtained by inverting the terms of the divisor, and then proceeding according to the rule for the multiplication of fractions. Thus the operation may be represented as follows:

$$\tfrac{5}{8}\div\tfrac{3}{7}=\tfrac{5}{8}\times\tfrac{7}{3}=\tfrac{35}{24}.$$

Hence, to divide one fraction by another, we have the following

RULE.

Invert the terms of the divisor, and then apply the rule for the multiplication of fractions.

This rule may be applied to every case in the division of fractions, by reducing mixed numbers to improper fractions, and regarding whole numbers as fractions having 1 for their denominator.

We may also apply the rule in the reduction of complex fractions to simple ones, since the numerator of any fraction may be regarded as a *dividend*, and the denominator as a *divisor.* If either or both of the terms of the complex fraction are mixed numbers, we may reduce them to improper fractions, and then apply the rule.

EXAMPLES.

1. Divide $\frac{17}{45}$ by $\frac{34}{15}$.

OPERATION.

$$\frac{17}{45}\div\frac{34}{15}=\frac{\cancel{17}}{\underset{3}{\cancel{45}}}\times\frac{\cancel{15}}{\underset{2}{\cancel{34}}}=\frac{1}{6}.$$ *Ans.* $\frac{1}{6}$.

2. Divide $5\frac{2}{8}$ by $7\frac{1}{3}$.

OPERATION.

$$5\frac{2}{8}\div7\frac{1}{3}=\frac{42}{8}\div\frac{21}{3}=\frac{\overset{\cancel{2}}{\cancel{42}}}{\underset{4}{\cancel{8}}}\times\frac{3}{\cancel{21}}=\frac{3}{4}.$$ *Ans.* $\frac{3}{4}$.

3. Divide $\frac{14}{26}\times\frac{13}{21}\times\frac{31}{45}$ by $\frac{3}{7}\times\frac{2}{5}\times\frac{31}{15}$.

OPERATION.

Terms of the Divisor Inverted.

$$\frac{\overset{7}{\cancel{14}}}{\underset{\cancel{2}}{\cancel{26}}}\times\frac{\cancel{13}}{\underset{3}{\cancel{21}}}\times\frac{\cancel{31}}{\underset{3}{\cancel{45}}}\times\frac{\cancel{7}}{3}\times\frac{5}{2}\times\frac{\cancel{15}}{\cancel{31}}=\frac{35}{54}.$$ *Ans.* $\frac{35}{54}$.

4. Reduce $\frac{2\frac{2}{3}}{5\frac{1}{7}}$ to a simple fraction.

OPERATION.

$$\frac{2\frac{2}{3}}{5\frac{1}{7}}=2\frac{2}{3}\div5\frac{1}{7}=\frac{8}{3}\div\frac{36}{7}=\frac{\overset{2}{\cancel{8}}}{3}\times\frac{7}{\underset{9}{\cancel{36}}}=\frac{14}{27}.$$ *Ans.*

5. Divide $\frac{16}{25}$ by $\frac{4}{25}$. *Ans.* 4.

6. Divide $\frac{15}{76}$ by $\frac{18}{42}$. *Ans.*

7. *Divide* $\frac{16}{21}$ *of* $\frac{14}{51}$ *by* $\frac{7}{8}$ *of* $\frac{3}{17}$. *Ans.* $\frac{4}{17}$.

8. Divide $152\frac{4}{13}$ by $18\frac{2}{6}$. *Ans.*

9. Divide $2345\frac{2}{5}$ by $5\frac{3}{7}$. *Ans.*

10. Divide 4500 by $11\frac{1}{9}$. *Ans.* 405.

11. Reduce $\frac{2\frac{5}{8}}{3\frac{1}{2}}$ to a simple fraction. *Ans.* $\frac{3}{4}$.

12. Reduce $\frac{10\frac{1}{2}}{5\frac{1}{4}}$ to a simple fraction. *Ans.* 2.

13. Reduce $\frac{19\frac{6}{11}}{9\frac{5}{9}}$ to a simple fraction. *Ans.* $\frac{645}{172}$.

14. Divide the sum of $\frac{3}{4}$, $\frac{1}{15}$, and $\frac{2}{5}$ by the sum of $\frac{1}{12}$, $\frac{1}{5}$, and $\frac{19}{20}$. *Ans.* $\frac{73}{74}$.

15. Divide the sum of $\frac{2\frac{3}{7}}{5\frac{4}{6}}$ and $\frac{1\frac{1}{2}}{5}$ by $\frac{7}{8}$. *Ans.* $1\frac{3}{49}$.

16. How many coat patterns, each containing $2\frac{3}{4}$ yards, may be cut from a piece of cloth containing 31 yards; and what quantity of cloth will remain?

Ans. 11 patterns, and $\frac{3}{4}$ of a yard will remain.

17. A hatter has 48 dollars which he wishes to expend in purchasing hats and caps. The hats are sold at $4\frac{1}{2}$ dollars, and the caps at $\frac{3}{5}$ of a dollar apiece. He wants to purchase as many hats as he can with his money, and lay out the balance in caps. How many hats and how many caps can he buy? *Ans.* 10 hats and 5 caps.

18. How many pounds of butter, at $14\frac{1}{2}$ cents per pound, should be exchanged for 29 yards of cloth, at $33\frac{1}{3}$ cents per yard? *Ans.* $66\frac{2}{3}$ pounds.

19. If $\frac{5}{8}$ of a ton of hay cost $11\frac{1}{4}$ dollars, what will a ton cost at the same rate? What will 40 tons cost?

Ans. A ton, 18 dollars; 40 tons, 720 dollars.

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20. A mason is to build a wall in 45 days; what part of the wall ought he to build in $23\frac{3}{4}$ days? *Ans.* $\frac{19}{36}$.

21. A man has 25 dollars. He pays $1\frac{3}{4}$ dollars per week for his board, and $1\frac{1}{2}$ dollars per week for the board of his son. In how many weeks will he have paid out the 25 dollars?

Ans. $7\frac{9}{13}$ weeks.

22. One boy leaves the town A, and another the town B, at the same time. The boy who leaves the town A, travels at the rate of $2\frac{1}{4}$ miles per hour, and the one leaving the town B, travels at the rate of $2\frac{5}{8}$ miles per hour. If the distance between the two towns is 195 miles, in how many hours will the boys meet each other, providing that each boy spends 48 hours in getting rest? *Ans.* 88 hours.

23. A merchant spends 21 dollars in purchasing slates and arithmetics. The arithmetics cost $\frac{3}{8}$ of a dollar apiece, and the slates, $\frac{1}{8}$ of a dollar apiece. He bought twice as many arithmetics as slates. How many of each did he buy?

Ans. 48 arithmetics, and 24 slates.

24. A merchant charged me 56 cents for $\frac{7}{8}$ of a yard of cloth. What did he charge per yard? *Ans.* 64 cents.

CHAPTER VI.

DECIMAL FRACTIONS AND FEDERAL MONEY.

DECIMAL FRACTIONS.

41. A *Decimal Fraction* is a fraction whose denominator is 1 with one or more ciphers annexed. It follows from this definition that decimal fractions arise from dividing the unit into 10 equal parts, and each of these parts into 10 other equal parts, and so on, forming periods of decimals which correspond to the periods of whole numbers.

42. To avoid the inconvenience of writing the denominators of decimal fractions, a period, called *the decimal point*, is used. The figure expressing *tenths* is placed at the right of the decimal point, the one expressing *hundredths*, occupies the next right hand place, and so on. Thus, $\frac{9}{10}$ is written .9, $\frac{45}{100}$ is written .45, which may be read *four tenths and five hundredths*, or simply, *forty five hundredths.* $\frac{7}{100}$ is written .07. As there are no *tenths* in this last decimal, a cipher is written before 7, in order that it might occupy the *place* of hundredths. Three thousandths, or $\frac{3}{1000}$, is written .003. Two hundred and forty-five thousandths, or $\frac{245}{1000}$, is written .245.

43. A cipher placed at the right of a decimal fraction does not alter its value. Thus, if we place a cipher at the right of .5, we have .50, but .50 is equal to .5, since each is equal to $\frac{1}{2}$. A cipher placed at the left of a decimal makes it 10 times as

small. Thus, if we place a cipher on the left of the decimal 5, we have .05, which is obviously 10 times as small as .5.

44. It is plain that decimals increase from the right hand towards the left by the same law as whole numbers do; that is, the value represented by any figure in a given number, is ten times as great as it would be, if this figure occupied the next right hand place. Hence, a whole number and a decimal fraction may be written together, the decimal point separating them. Thus, 24 and .045 may be written 24.045.

45. The following table will enable the student to read and write decimals. In reading decimals, *read the decimal as if it were a whole number, and add the name of the last decimal figure.* Thus, 0.045 is read forty-five *thousandths.* In writing decimals, observe that *the number of decimal figures must equal the number of ciphers in the denominator.*

WHOLE NUMBERS.							DECIMALS.									
&c.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.	Tenths.	Hundredths.	Thousandths.	Ten-Thousandths.	Hundred-Thousandths.	Millionths.	Ten-Millionths.	Hundred-Millionths.	Billionths.	&c.
	4	2	3	4	5	6	.7	2	1	5	6	8	9	0	5	

EXAMPLES.

Read the following decimals:

1.	0.0456	7.	0.04568
2.	0.4568	8.	2.6567
3.	0.025	9.	0.0005
4.	6.85402	10.	0.002007
5.	8.04053	11.	0.4000004
6.	2.36854	12.	0.4004004.

Write the following decimals in figures:

1. Twenty-five thousandths.

2. Two thousand eight hundred and forty-five ten-thousandths.

3. Seven, and five hundred millionths.

4. Forty-five, and three hundred and seventy-five ten-thousandths.

5. Five, and two hundred and sixty-three millionths.

6. Nine hundred and eighty, and four thousandths.

7. Two, and eighty-five billionths.

8. Two hundred and seventy-four tenths.

9. Eight thousand two hundred and thirty-two thousandths.

10. Four hundred and fifty-two hundred-thousandths.

11. Sixty-five, and five hundred and twenty-one thousandths.

12. Eighty-two, and sixty-five billionths.

ADDITION OF DECIMALS.

46. In the addition of whole numbers, we place the units of the same order in the same vertical column, and since de-

cimals increase from the right hand towards the left according to the same law by which whole numbers do, it will also be convenient to arrange them in a similar manner; that is, tenths under tenths, hundredths under hundredths, and so on. The decimal points will then fall under each other.

Let it be required to add the decimals 0.458 and 0.087.

Operation.

$$\begin{array}{r} 0\,.\,4\,5\,8 \\ 0\,.\,0\,8\,7 \\ \hline 0\,.\,5\,4\,5 \end{array}$$

Having properly arranged the decimals, we commence at the column of thousandths to add. We say that 8 thousandths and 7 thousandths are 15 thousandths, which are equal to 10 thousandths plus 5 thousandths. We set down the 5 thousandths in the place of thousandths, and observe that 10 thousandths are equal to 1 hundredth. This 1 hundredth we add to the first figure 8, in the column of hundredths and obtain 9 hundredths, which we add to 5 hundredths and obtain 14 hundredths, which are equal to 10 hundredths plus 4 hundredths. We set down the 4 hundredths under the column of hundredths, and then observe that 10 hundredths are equal to 1 tenth, which we add to 4 in the next column and obtain 5 tenths, which we set under the column of tenths. Hence, the sum required is 0.545. Hence, for the addition of decimal fractions, we have the following

RULE.

Set down the fractions in such a manner that the decimal points may fall directly under each other. Then add as in whole numbers, and in the sum place the decimal points so that it shall be directly under the other decimal points.

EXAMPLES.

1. What is the sum of 78.345, 364.504, 27.0452, 504.05, and 3248.063?

OPERATION.

78.345
364.504
27.0452
504.05
3248.063
4222.0072 *Ans.*

2. What is the sum of 0.00045, 256.4, 48.3056, 6468.045, 0.0045, and 24.982? *Ans.* 6797.73755.

3. What is the sum of 256.456, 38.256, 85.235, 24.565, 2384.25, and 0.0456? *Ans.* 2788.8076.

4. What is the sum of 456.205, 0.4585, 785.645, 0.0982, and 396.458? *Ans.* 1638.8647.

5. What is the sum of 25604.25, 0.045, 28.536, 560.231, 0.04582, and 0.45? *Ans.* 26193.53282.

6. What is the sum of five thousandths, twenty-five millionths, eight hundredths, seventeen ten-thousandths, seventy millionths, and two hundred and thirty-four thousandths? *Ans.* 0.320795.

7. What is the sum of forty-two tenths, eighty-seven thousandths, twenty-six ten-thousandths, seven hundred and ninety-eight one-hundred-thousandths, and two hundred and thirty-two? *Ans.* 236.29758.

8. What is the sum of 2456.045, 0.417, 0.245, 25.63, and 582.4158? *Ans.* 3064.7528.

9. What is the sum of 175.25, 381.26, 516.045, 2384.63, and 3417.815? *Ans.* 6875.

10. What is the sum of 2451.382, 0.456, 645.041, 3851.2502, and 0.457? *Ans.* 6948.5862.

11. What is the sum of 164.58, 384.421, 75.604, 585.21, 3891.615, and 0.451? *Ans.* 5101.881.

12. What is the sum of 1685.04, 904.52, 0.56, 703.8904, 6503.7123, and 7836.202? *Ans.* 17633.9247.

13. What is the sum of 256.45, 2309.041, 1785.061, 3201.531, 7850.6031, 0,458, and 2034.503?

Ans. 17437.6471.

SUBTRACTION OF DECIMALS.

47. For the subtraction of decimal fractions, we have the following

RULE.

Place the subtrahend under the minuend, observing to have the decimal point of the former directly under that of the latter, and then subtract as in whole numbers.

EXAMPLES.

1. From 28.25 subtract 20.0045.

OPERATION.

$$\begin{array}{r} 28.2500 \\ \underline{20.0045} \\ 8.2455 \end{array} \text{ \textit{Ans.}}$$

In examples like this, in which there are not as many decimal figures in the minuend as in the subtrahend, we may annex ciphers to the minuend, or conceive them to be annexed.

2. *From* 285.045 subtract 251.67. Ans. 33.375.

3. From 7856.023 subtract 285.067. *Ans.* 7570.956.

4. From 2382.0405 subtract 204.067. *Ans.* 2177.9735.

5. From 6782.0502 subtract 45.000402.
Ans. 6737.049798.

6. From 214.81 subtract 4.90132. *Ans.* 209.90868.

7. From 2714 subtract 0.916. *Ans.* 2713.084.

8. From eighty-nine tenths, subtract twenty-seven millionths.
Ans.

9. From two hundred and seventy-five, subtract two hundred and seventy-five thousandths. *Ans.*

10. Find the difference between two hundred and eighty-seven ten-thousandths and the same number of thousandths.
Ans.

11. Find the difference between 45 and 45 hundredths.
Ans.

12. From 25805.14 take 1 millionth. *Ans.*

MULTIPLICATION OF DECIMALS.

48. Let it be required to multiply 0.384 by 0.251. We may observe that

$$0.384=\tfrac{384}{1000}, \text{ and } 0.251=\tfrac{251}{1000};$$

therefore, $0.384\times0.251=\frac{384}{1000}\times\frac{251}{1000}=\frac{96384}{1000000}$, by the rule for the multiplication of common fractions. But, according to the notation of decimals,

$$\tfrac{96384}{1000000}=0.096384;$$

hence, 0.096384 is the product required, expressed decimally. *We see that the number* of decimal places in this product, is

equal to the number of decimal places in the two factors. If we take any other two factors, we shall find that the number of decimal places in their product is equal to the number of decimal places in the two factors. Hence, for the multiplication of decimals, we have the following

RULE.

Multiply as in whole numbers, and from the right of the product, point off as many decimal places as there are in both factors. If the number of decimal places in the product is not equal to the number of decimal places in both factors, prefix ciphers to supply the deficiency.

EXAMPLES.

1. Multiply 79.347 by 23.15. *Ans.* 1836.88305.
2. Multiply 0.63478 by 0.8204. *Ans.* 0.520773512.
3. Multiply 0.385746 by 0.00464. *Ans.* 0.00178986144.
4. Multiply 0.0045 by 0.8325. *Ans.* 0.00374625.
5. Multiply 0.234 by 234. *Ans.* 54.756.
6. Multiply 0.0642 by 23.04. *Ans.* 1.479168.
7. Multiply 0.302 by 0.7854. *Ans.* 0.2371908.
8. Multiply 23.045 by 2.0413. *Ans.*

9. Multiply seventy-three thousandths by three hundred and forty-seven ten-thousandths. *Ans.*

10. Multiply 275 by two hundred and seventy-five thousandths. *Ans.*

11. If a man can walk 42.37 miles in one day, how many miles can he walk in 43 days? *Ans.* 1821.91 miles.

12. According to Prof. Pierce, the mean velocity of sound

in the atmosphere, is about 1090 feet per second. What is the distance of a hill on which a cannon is fired, if 8.75 seconds elapse between the flash and report? *Ans.*

13. What is the price of 16 boxes of raisins, each containing 9.4 pounds, at 9.25 cents per pound? *Ans.*

14. Multiply forty-eight thousandths by forty-eight thousand. *Ans.* 2304.

49. To multiply a decimal fraction by 10, 100, 1000, &c., remove the decimal point as many places to the right as there are ciphers in the multiplier. The product of 0.2457 by 100 is 24.57. The product of 27.8056 by 1000 is 27805.6.

Multiply the first four following numbers by 100, and the second four by 1000:

1.	945.6805	5.	85.0056
2.	302.585	6.	252.045
3.	2.01	7.	702.3582
4.	5.85065	8.	582.45

9. Multiply 3.0000052 by 100000; by 10000000. *Ans.*

DIVISION OF DECIMALS.

50. In the multiplication of decimals, it has been observed that the number of decimal places in the product is equal to the number of decimal places in both factors. As the dividend is equal to the product of the divisor and quotient, it follows that the number of decimal places in the dividend is equal to the number in both divisor and quotient. Therefore the number of decimal places in the quotient is equal to the number in the dividend diminished by the number in the divisor. Hence, for the division of decimals, we have the following

RULE.

Divide as in whole numbers; and point off from the right hand of the quotient as many decimal places, as the decimal places in the dividend exceed those in the divisor. When the number of figures in the quotient is less than the number of decimal places in the dividend, diminished by the number in the divisor, supply the deficiency by prefixing ciphers.

To divide by 10, 100, 1000, &c., remove the decimal point in the dividend as many decimal places to the left as there are ciphers in the divisor. The quotient of 0.04 by 10, is 0.004. The quotient of 24.56 by 1000 is 0.02456.

EXAMPLES.

1. Divide 0.625 by 2.5. *Ans.* 0.25.
2. Divide 9.1125 by 4.5. *Ans.* 2.025.
3. Divide 14 by 0.7854.* *Ans.* 17.825+.
4. Divide 2175.68 by 100. *Ans.* 21.7568.
5. Divide 0.8727587 by 0.162. *Ans.* 5.38739+
6. Divide 45.5 by 2100. *Ans.* 0.0216+.
7. Divide 47.655 by 4.5. *Ans.* 10.59.
8. Divide 269.0625 by 14.35. *Ans.* 18.75.
9. Divide 555 by 0.0037. *Ans.* 150000.
10. There are 5.5 yards in a rod, and 1760 yards in one mile; how many rods are there in a mile? *Ans.* 320.

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* NOTE.—When the dividend is not exactly divisible by the divisor, we may annex ciphers to the dividend, and carry on the division to any extent required.

11. If one ton of hay cost 9.75 dollars, how many tons can be bought for 204.75 dollars ? *Ans.* 21 tons.

12. If a man walk 3.75 miles in an hour, how many hours will he require to walk 787.5 miles? *Ans.* 210 hours.

13. How many revolutions will a wheel that is 14.25 feet in circumference make in going a distance of 5280 feet.

Ans. 370.52 times, nearly

REDUCTION OF COMMON FRACTIONS TO DECIMALS.

51. For the reduction of common fractions to decimal fractions we have the following

RULE.

Annex ciphers to the numerator for decimal figures, and then divide the numerator by the denominator. Continue the division till a quotient is obtained which is sufficiently exact.

EXAMPLES.

1. Reduce $\frac{3}{4}$ to its equivalent decimal fraction.

OPERATION.

$$\begin{array}{r} 4)3.00 \\ \hline 0.75 \end{array} \quad \textit{Ans.}$$

Hence, we observe that 3, the numerator, is equal to 30 *tenths*, or 300 *hundredths*, and $\frac{1}{4}$ of 300 hundredths is 75 hundredths. Hence $\frac{3}{4}=0.75$.

2. Reduce $\frac{5}{8}$ to a decimal fraction. *Ans.* 0.625.

3. Reduce $\frac{17}{25}$ to a decimal fraction. *Ans.* 0.68.

4. Reduce $\frac{19}{40}$ to a decimal fraction. *Ans.* 0.475.

5. Reduce $\frac{56}{45}$ to a decimal fraction. *Ans.* 1.244+.

6. Reduce $\frac{4}{7}$ to a decimal fraction. *Ans.* 0.571+.

7. Reduce $\frac{5}{17}$ to a decimal fraction. *Ans.* 0.2941+.

8. Reduce $\frac{9}{35}$ to a decimal fraction. *Ans.* 0.257+.

9. Reduce $\frac{17}{51}$ to a decimal fraction. *Ans.* 0.333+.

10. Reduce $\frac{19}{152}$ to a decimal fraction. *Ans.* 0.125.

11. Reduce $\frac{1}{4}$, $\frac{1}{5}$, $\frac{3}{8}$, $\frac{2}{3}$, $\frac{5}{6}$ and $\frac{9}{16}$ to decimal fractions.
Ans.

12. Reduce $\frac{49}{50}$ to a decimal fraction. *Ans.*

REDUCTION OF A DECIMAL TO A COMMON FRACTION.

52. It is plain, that for the reduction of a decimal fraction to a common fraction, we have the following

RULE.

Erase the decimal point, and write under the decimal figures a unit, followed by as many ciphers as there are decimal places.

EXAMPLES.

1. Reduce 0.275 to a common fraction.

OPERATION.

$$0.275=\frac{275}{1000}=\frac{11}{40}.$$

Reduce the following decimals to common fractions:

1. 0.035. *Ans.* $\frac{7}{200}$. 2. 0.165. *Ans.* $\frac{33}{200}$.

3. 0.644. *Ans.* $\frac{161}{250}$.
4. 0.026. *Ans.* $\frac{13}{500}$.
5. 0.375. *Ans.* $\frac{3}{8}$.
6. 0.76482. *Ans.* $\frac{38241}{50000}$.
7. 0.4375. *Ans.* $\frac{7}{16}$.
8. 2.25. *Ans.* $\frac{9}{4}$.

FEDERAL MONEY.

53. Federal Money is the currency of the United States. The unit in this currency is called a *dollar.* The *tenth* part of this unit is called a *dime;* the *tenth* part of the dime is called a *cent;* and the tenth of the cent is called a *mill.* We may conceive the unit, then, to be divided into *ten* equal parts, each of these parts into *ten* other equal parts, and so on. Hence, Federal Money is based on the *Decimal System of Notation,* and to this circumstance it owes its great simplicity.

The denominations of Federal Money are Eagles, Dollars, Dimes, Cents, and Mills. The Eagle is a gold coin, the Dollar and Dime are silver coins, and the Cent is a copper coin. The Mill is not coined. Gold and silver coins contain $\frac{9}{10}$ of pure metal and $\frac{1}{10}$ alloy. The alloy of gold is composed of silver and copper. The silver must not exceed the copper in weight. The alloy of silver is pure copper. Gold and silver thus alloyed are called *standard.* The Eagle contains 258 grains of standard gold, the dollar 412½ grains of standard silver, and the cent 168 grains of pure copper.

TABLE OF FEDERAL MONEY.

10 Mills	make	1 Cent,	marked	*ct.*
10 Cents	"	1 Dime,	"	*d.*
10 Dimes	"	1 Dollar,	"	\$.*
10 Dollars	"	1 Eagle,	"	E.

* This symbol is probably a contraction of U. S. (United States) the U being placed on the S.

In addition to the coins which we have mentioned, there are the Double Eagle, Half-Eagle, Quarter-Eagle, and Dollar, which are made of gold ; and the half-dollar, quarter-dollar, half-dime, and the three-cent piece, which are made of silver.

Every sum in Federal money is expressed in dollars, cents, and mills, and in reading any sum, we do not mention any other denominations. Thus, $25.46 is not read two eagles, 5 dollars, 4 dimes, and 6 cents, but *twenty-five dollars and forty-six cents.**

54. Federal money being based on the Decimal system of Notation, all operations in this currency, are performed by means of the rules in Decimal Fractions.

EXAMPLES.

1. What is the amount of $194.04, $296.45, $384, $1.0625, and $3842? *Ans.* $4717.5525.

2. A person received $323.45, and then paid $25.63 for books, 35.45 for a suit of clothes, and $125 for a horse. How much did he have left? *Ans.* $137.37.

3. A farmer sold his wheat for $325.63, his pork for $83.43, his butter for $95.91, and his barley for $115.21. How much did he receive for these commodities? *Ans.* $620.18.

4. A farmer sold 425 bushels of wheat at $1.25 per bushel. What did he receive for his wheat? *Ans.* $531.25.

5. What is the cost of 45 pounds of tea at 43 cents per pound? *Ans.* $19.35.

NOTE.—Since cents are decimals of a dollar, we may read $25.46, *twenty-five dollars, and forty-six hundredths of a dollar.*

6. How many hats at $2.75 a piece, can be bought for $68.75? *Ans.* $25.

7. A laborer engages to work for 18 months, at $13.75 per month; what will his wages amount to? *Ans.* $247.50.

8. If a man can earn $18.75 in 10 days, how many dollars can 45 men earn in the same time? *Ans.* $843.75.

9. The coal from the anthracite coal-mines on the Lehigh River, Pa., is conveyed from the mines by means of a self-acting railway for 8 miles down a declivity, from 100 to 140 feet per mile, at a cost of about 4 cents per ton. What will be the cost of obtaining 2465 tons from the mines? *Ans.* $98.60.

10. What is the cost of 27 yards of satin at 87½ cents per yard?* *Ans.* $23.625.

11. What is the cost of 217 bushels of wheat at $1.12½ per bushel? *Ans.* $244.125.

12. What is the cost of 2384 yards of broadcloth at $4¾ per yard? *Ans.* $11324.

13. A merchant bought 28.5 yards of cloth, for which he paid $78.375. At what rate per yard must he sell it, in order to make 35 cents on each yard? *Ans.* $3.10.

14. What must be the increase in the weight of a pig for 42 days, to defray the expense of keeping, when corn sells at 56 cents per bushel, and pork at 12 cents per pound, and he consumes ⅛ of a bushel of corn and 3 cents worth of other food each day? *Ans.* 35 pounds.

15. A farmer fatted an ox for which he was offered $50. He continued to feed the ox for 60 days, and the ox consumed each day 12 quarts of meal, worth 2 cents per quart, and 10

* NOTE.—Observe that 87½ cents=$0.875.

cents worth of other food. At the end of this time he sold the ox for $70. Did the farmer lose or gain by fatting his ox?

Ans. He lost 40 cents.

16. How many bushels of wheat, at $1.25 per bushel, must be exchanged for 35 yards of cloth at $3.75 per yard?

Ans. 105 bushels.

17. A farmer owes a merchant $53.82, and the merchant agrees to take his pay in butter and cheese, of each the same number of pounds. The butter is worth 15 cents per pound, and the cheese 8 cents. How many pounds of each must the farmer let the merchant have? *Ans.* 234.

18. What is the cost of 13.75 tons of hay at $6.75 per ton? *Ans.*

19. What is the cost of 75 window springs at 11 cents a piece? *Ans.*

20. What is the cost of 223 door butts at $4\frac{1}{2}$ cents a piece?

Ans.

21. If 19 cords of wood cost $61.75, what is the price per cord? *Ans.* $3.25.

22. What is the cost of 15.825 cords of wood, at $2.75 per cord? *Ans.* $43.51875.

23. What is the cost of 36 saws at 1.87\frac{1}{2}$ a pieec?

Ans. $67.50.

24. In 1852, the whole number of classical students in the several academies in the State of New York, was 19552, and in 1853, this number was 20920. The amount which is annually granted to these institutions is $40000. What is the rate per student for these two years?

Ans. In 1852, $2.04; in 1853, $1.91.

25. At Rochester, New York, the mean temperature for the

several months in the year 1851, was as follows: January, 28°.5, February, 32°.2, March 35°.9, April 43°.2, May 55°.9, June, 65°.6, July 70°.3, August 67°.7, September 62°.2, October, 50°.3, November 35°.8, December 24°.0. What was the annual mean temperature?* *Ans.* 47°.6$\frac{1}{3}$.

26. In 1851, the amount of rain which fell at Flatbush, New-York, during the several months of the year, was as follows: January 3.09 inches, February 3.03 inches, March 3.55 inches, April 3.26 inches, May 3.90 inches, June 3.52 inches, July 3.21 inches, August 4.41 inches, September 3.09 inches, October 3.39 inches, November 3.24 inches, December 3.74 inches. What was the whole amount of rain which fell during the year, and what was the monthly average?

Ans. Whole amount, 41.43 inches; monthly average, 3.45$\frac{1}{4}$ inches.

27. What is the cost of 3250 bricks, at $4.25 per thousand?† *Ans.* $13.8125.

28. What is the cost of 2346 feet of lumber at $1.75 cents per hundred? *Ans.* $41.055.

29. What is the cost of 1230 pounds of beef at $7.25 per hundred? *Ans.* $89.175.

30. If a man drinks 3 glasses of rum each day, at 3 cents a glass, how much would he spend in 45 years, each containing 365 days? *Ans.* 1478.35.‡

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* The mean of any number of quantities is found by dividing their sum by the number of quantities.

† See Article 50. First get the cost of one brick by dividing by 1000.

‡ Note.—If we take into consideration, yearly compound interest, at 7 *per cent, we shall* find the rum, which he would drink, would cost

31. If a man spend 12 cents a day for cigars, what amount will he spend in 30 years, of 365 days? *Ans.* \$1314.00.

55. Many of the computations in Federal Money, may be more readily made by making use of the aliquot parts of a dollar. Any aliquot part of any number, or quantity, is an exact part of that number or quantity. Thus, one of the aliquot parts of \$1 is 50 cents, since 50 cents$=\$\frac{1}{2}$. Some of the aliquot parts of a dollar are given in the following

TABLE.

5 cts. $= \$\frac{1}{20}$	25 cts. $= \$\frac{1}{4}$
$6\frac{1}{4}$ cts. $= \$\frac{1}{16}$	$31\frac{1}{4}$ cts. $= \$\frac{5}{16}$
$8\frac{1}{3}$ cts. $= \$\frac{1}{12}$	$37\frac{1}{2}$ cts. $= \$\frac{3}{8}$
10 cts. $= \$\frac{1}{10}$	50 cts. $= \$\frac{1}{2}$
$16\frac{2}{3}$ cts. $= \$\frac{1}{6}$	$62\frac{1}{2}$ cts. $= \$\frac{5}{8}$
$18\frac{3}{4}$ cts. $= \$\frac{3}{16}$	$66\frac{2}{3}$ cts. $= \$\frac{2}{3}$
20 cts. $= \$\frac{1}{5}$	$87\frac{1}{2}$ cts. $= \$\frac{7}{8}$

EXAMPLES.

1. What is the cost of 240 yards of cloth at $87\frac{1}{2}$ cents per yard?

The cost of 240 yards at \$1 per yard, is . . . 240
The cost of 240 yards at $12\frac{1}{2}$ cts.$=\$\frac{1}{8}$ per yard, is 30

Ans. \$210

If we subtract the cost of the cloth at $12\frac{1}{2}$ cents, or $\frac{1}{8}$ of a dollar, from the cost at 1 dollar per yard, we shall obviously have the cost of the cloth at $87\frac{1}{2}$ cents per yard, which is \$210.

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him \$9387.12. For making this computation, the student is referred to *my larger work* on Algebra, Problem V., page 251.

2. What is the cost of 2385 bushels of wheat, at 75 cents per bushel? *Ans.* \$1788.75.

3. What is the cost of 4560 bushels of potatoes at 31¼ cents per bushel? *Ans.* \$1425.

4. What is the cost of 840 pounds of hay, at the rate of \$1.12½ for 100 pounds? *Ans.* \$9.45.

5. What is the cost of 40 pieces of calico, each piece containing 30 yards, at 6¼ cents per yard? *Ans.* \$75.

6. What is the cost of 4840 feet of lumber, at \$8.75 per thousand feet? *Ans.* \$42.35.

7. What is the cost of 480 bushels of barley, at 56¼ cents per bushel? *Ans.* \$270.

8. What is the cost of 12 sets of knives and forks, at \$1.43¾ per set? *Ans.* \$17.25.

9. What is the cost of 16 pounds of tea, at 62½ cents per pound? *Ans.* \$10.

10. What is the cost of 24 sets of gate hinges, at 75 cents per set? *Ans.* \$18.

11. What is the cost of 48 Algebras, at 62½ cents per copy? *Ans.* \$30.

12. How many bushels of oats, at 37½ cents per bushel, can be purchased for \$36.87½? *Ans.* 98⅓.

13. How many hats at 87½ cents per hat, can be purchased for \$35? *Ans.* 40.

14. What is the cost of 48 gallons of molasses, at 33⅓ cents per gallon? *Ans.* \$16.

15. What is the cost of 540 pounds of butter, at 18¾ cents per pound? *Ans.* \$101.25.

16. What is the cost of 688 feet of lumber, at 7s. 6d.* per hundred? *Ans.* \$6.45.

17. What is the cost of 18 gross of table hinges, at 3s. 6d. per gross? *Ans.* \$7.87½.

18. What is the cost of 24 yards of cloth, at 5s. 9d. per yard? *Ans.* \$17.25.

19. What is the cost of 450 bushels of wheat at 9s. 6d. per bushel? *Ans.* \$534.37½.

20. What is the cost of 48 hats, at 2s. 9d. per hat? *Ans.* \$16.50.

21. What is the cost of 1440 yards of linen, at 1s. 6d. per yard? *Ans.* \$360.

22. What is the cost of 840 pair of gloves, at 2s. 4d. per pair? *Ans.* \$326.66⅔

23. What is the cost of 250 bushels of oats, at 1s. 8d. per bushel? *Ans.* \$69.44⅘.

24. What is the cost of 48 glass lamps, at 3s. 4d. per lamp? *Ans.* \$26.66⅔.

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* NOTE.—"7s. 6d.," is read 7 *shillings and* 6 *pence.* A shilling, in New York, is equal to $\frac{1}{8}$ of a dollar, and a penny is equal to $\frac{1}{12}$ of a shilling. In New England, a shilling is equal to $\frac{1}{6}$ of a dollar, and a penny is equal to $\frac{1}{12}$ of a shilling The law requires that accounts be kept in Federal Money, but tradesmen frequently sell their goods at prices given in shillings and pence. From Example 16 to 20 inclusive, the prices are given in N. Y. currency. The prices in the remaining examples are given in N. E. currency. See Reduction of Currencies.

In solving the 16th example, we may observe, that, at \$1 per hundred, or 1 *cent per foot*, the cost must be \$6.88; and at 6d., which is $\frac{1}{16}$ of 1 dollar, the cost is $\frac{1}{16}$ of \$6.88=\$0.43. If we subtract the cost at 6d. per hundred from the cost at 8s.=\$1, we shall have the cost at 7s.

The pupil can see for himself what are the aliquot parts of a dollar.

CHAPTER VIII.

DENOMINATE NUMBERS.

56. A *Denominate Number* is an expression for a quantity by means of *different* measuring units which are related to each other in some particular manner. The term *foot* is an expression for a certain length, and the term *inch* is used to denote the *one-twelfth* part of the foot. Then a line that is twenty-seven inches long may be denoted by the number 27, in which the measuring unit is an *inch*, or we may have, as an expression for the length of the line, 2 *feet and* 3 *inches*. This last expression is a denominate number, and it is written, 2ft. 3in., or the abbreviations, *ft.* and *in.*, may be written over the numerals. Such an expression may also be called a *Denominate Quantity*.

Before the pupil can make computations in Denominate Numbers, he must commit to memory the following principal tables of money, weights, and measures:

ENGLISH OR STERLING MONEY.

57. English or Sterling Money is the currency of *Great Britain*.

TABLE.

4 farthings (*qr.* or *far.*)	make	1 penny, marked	*d.*
12 pence	"	1 shilling, "	*s.*
20 shillings	"	1 pound, or sovereign	£

The pound is a gold coin, called a sovereign. It consists of 22 parts of pure gold, and 2 parts of copper. Its value is $4.84.

TROY WEIGHT.

58. This weight is used in weighing gold, silver, and liquids. The Troy ounce is adopted as the standard at the United States Mint.*

TABLE.

24 grains (*grs.*)	make	1 pennyweight,	marked	*pwt.*
20 pennyweights	"	1 ounce,	"	*oz.*
12 ounces	"	1 pound,	"	*lb.*

AVOIRDUPOIS WEIGHT.

59. This weight is used in weighing all coarse and drossy articles, such as sugar, tea, butter, grains, meat, &c.; and all metals, except gold and silver.†

TABLE.

16 drams (*dr.*)	make	1 ounce,	marked	*oz.*
16 ounces	"	1 pound,	"	*lb.*
25 pounds	"	1 quarter,	"	*qr.*
4 quarters	"	1 hundred weight		*cwt.*
20 hundred weight	"	1 ton,	"	*T.*

* NOTE.—The Troy pound is taken as the standard unit of weight by the United States government. It is equal to the weight of 22.7944 cubic inches of distilled water, at its maximum density, the barometer standing at 30 inches.

† NOTE.—The weight of the avoirdupois pound is equal to the weight of 7000 grains Troy, or to 27.7274 cubic inches of distilled water, the temperature being 62° Fahrenheit, and the barometer standing at 30 inches.

The quarter is generally estimated at 28 pounds, in measuring

APOTHECARIES' WEIGHT.

60. This weight is used by apothecaries and druggists in *mixing* their medicines. The pound is equal to the pound Troy weight, but the ounce is divided and subdivided into drams, scruples, and grains.

TABLE.

20 grains (*gr.*)	make 1 scruple,	marked	℈.
3 scruples	" 1 drachm,	"	ʒ.
8 drachmns	" 1 ounce,	"	℥.
12 ounces	" 1 pound,	"	lb.

LONG MEASURE.

61. This measure is used to measure distances. The *linear unit of measure* adopted by the United States is the yard, copied from the Imperial yard of Great Britain.*

TABLE.

12	inches (*in.*)	make 1 foot,	marked	*ft.*
3	feet	" 1 yard,	"	*yd.*
$5\frac{1}{2}$	yards	" 1 rod,	"	*rd.*
40	rods	" 1 furlong	"	*fur.*
8	furlongs	" 1 mile,	"	*mi.*
3	miles	" 1 league,	"	*L.*
$69\frac{1}{6}$ 60	statute miles, or geographical miles	" 1 degree,	"	*deg.* or °

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some of the coarser articles, but the laws of the United States require that the quarter be estimated at 25 pounds.

* NOTE.—This yard is equal to $\frac{360000}{391393}$ of the length of a pendulum vibrating seconds, in a vacuum, in London.

CLOTH MEASURE.

62. This measure is used for measuring cloth, and all articles which are bought and sold by the yard.

TABLE.

$2\frac{1}{4}$ inches (*in.*)	make	1 nail,	marked	*na.*
4 nails	"	1 quarter,	"	*qr.*
4 quarters	"	1 yard,	"	*yd.*
3 quarters	"	1 Ell Flemish,	"	*E. Fl.*
5 quarters	"	1 Ell English,	"	*E. E.*
6 quarters	"	1 Ell French,	"	*E. Fr.*

SQUARE MEASURE.

63. This measure is used for measuring surfaces; as land, painting, plastering, &c. The unit of measure, in measuring surfaces, is always a *square* surface. A square is a figure having four sides, which are equal to each other, and having its angles right angles.

The values of the different denominations in Square and Cubic Measure are found by means of the standards in Long Measure.*

* NOTE.—In measuring land, surveyors employ a chain that is four rods, or 66 feet, long. The chain is divided into 100 equal parts, called links. The following are the denominations:

7.72 inches (*in.*)	make	1 link,	marked	*l.*
100 links, or 4 rods	"	1 chain,	"	*ch.*
16 square rods	"	1 square chain,	"	*sq. ch.*
10 square chains	"	1 acre,	"	*A.*

TABLE.

144 square inches (*sq. in.*)	make	1 square foot,	marked	*sq. ft.*
9 square feet	"	1 square yard	"	*sq. yd.*
$30\frac{1}{4}$ square yards	"	1 rod or pole,	"	*P.*
40 square rods	"	1 rood,	"	*R.*
4 roods	"	1 acre,	"	*A.*
640 acres	"	1 square mile,	"	*M.*

CUBIC MEASURE.

64. This measure is used in measuring solid bodies, or any thing which has *length*, *breadth*, and *thickness*. The unit of measure in measuring solids, is a cube. A cube is a solid which is bounded by six equal squares. When these equal squares are square feet, then the cube is called a *cubic foot;* when they are square yards, the cube is called a cubic yard, and so on.

TABLE.

1728 cubic inches (*cu. in.*)	make	1 cubic foot,	marked	*cu. ft.*
27 cubic feet	"	1 cubic yard,	"	*cu. yd.*
40 feet of round timber, or 50 feet of hewn timber*	"	1 ton or load,	"	*Ton.*
16 cubic feet	"	1 cord foot,	"	*c. ft.*
8 cord feet, or 128 cubic feet	"	1 cord,	"	*C.*

* NOTE.—The ton is thus estimated in England, but in the United States the ton is generally, if not always, estimated at 40 cubic feet for both round and hewn timber. A ton of round timber is such a quantity as will, when hewn, be equal to the weight of 50 cubic feet of hewn timber. If the diameter and length of a log are such that it will, when hewn, contain 40 cubic feet, then it is called a ton of round timber.

WINE MEASURE.

65. This measure is used in measuring all liquids except ale, beer, and milk.*

TABLE.

4 gills (*gi.*)	make 1 pint,	marked	*pt.*
2 pints	" 1 quart,	"	*qt.*
4 quarts	" 1 gallon,	"	*gal.*
31½ gallons	" 1 barrel,	"	*bar.*
2 barrels, or 63 gallons	" 1 hogshead,	"	*hhd.*
2 hogsheads	" 1 pipe,	"	*pi.*
2 pipes	" 1 tun,	"	*tun.*

BEER MEASURE.

66. This measure is used in measuring ale, beer, and milk.†

TABLE.

2 pints (*pt.*)	make 1 quart,	marked	*qt.*
4 quarts	" 1 gallon,	"	*gal.*
36 gallons	" 1 barrel,	"	*bar.*
1½ barrels	" 1 hogshead,	"	*hhd.*

* NOTE.—The standard which has been adopted by the United States, for measuring liquids, is the Wine Gallon, which contains 8.3389 pounds avoirdupois of distilled water, at the temperature of 39°.83 Fahrenheit, the barometer standing at 30 inches. It contains 231 cubic inches.

† NOTE.—The beer gallon contains 282 cubic inches of distilled water, at the temperature of 39°.83, the barometer standing at 30 inches.

ale,

DRY MEASURE.

67. This measure is used in measuring grain, salt, coal, sand, &c.*

TABLE.

2 pints (*pi.*)	make	1 quart,	marked	*qt.*
8 quarts	"	1 peck,	"	*pk.*
4 pecks	"	1 bushel,	"	*bu.*
32 bushels	"	1 chaldron,	"	*ch.*

TIME.

68. By the revolution of the earth about the sun, and its axis, time is naturally divided into years and days. This and the other denominations are given in the following

TABLE.

60 seconds	make	1 minute,	marked	*min.*
60 minutes,	"	1 hour,	"	*hr.*
24 hours,	"	1 day,	"	*d.*
7 days	"	1 week,	"	*wk.*
52 weeks, 1 d. 6 hr.,† or 12 calendar months	"	1 year,	"	*y.*

In business transactions, 30 days are taken for a month.

* Note.—The standard in this measure is the bushel, which contains 77.6274 pounds avoirdupois of distilled water, at the temperature of of 39°.83 Fahrenheit, the barometer standing at 30 inches. The capacity of this bushel measure is 2150.4 cubic inches, very nearly.

† Note.—It is shown in treatises on astronomy, that the earth makes a revolution about the sun in 365 days, 5 hours, 48 minutes, and 48 seconds; that is, the year contains $365\frac{1}{4}$, days, nearly. But it is more convenient to have the year consist of a whole number of

The following are the names of, and the number of days in, each of the months:

Number of the Month.	Name of the Month.	Number of Days.
1	January,	31.
2	February,	28.
3	March,	31.
4	April,	30.
5	May,	31.
6	June,	30.
7	July,	31.
8	August,	31.
9	September,	30.
10	October,	31.
11	November,	30.
12	December,	31.

The following table shows the number of days from any day of one month to the same day in an other month in the same

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days; hence three successive years are made to consist of 365 days each, and the fourth of 366 days. The year which contains 366 days is called a *Bissextile* year. It is also called *Leap* year. The day which is added to 365 days to form a Leap year, is called the *Intercalary* day. Every year, the number of which is exactly divisible by 4, is a bissextile. The intercalary day is added to February.

Since the year is not quite equal to $365\frac{1}{4}$, it follows that we commit an error by making every fourth a leap year. To correct this error, three intercalary days are omitted every four hundred years. It may be seen, by a short calculation, that the error is very nearly corrected by this means. The intercalary days are omitted in those centurial years, the numbers of which are not divisible by 400. Thus the years 1700, 1800, 1900, 2100, are common years, instead of being Leap years.

year. It is constructed upon the supposition that February contains 28 days; hence, in Leap years, one day must be added to the time found from the table, if February constitutes a part of the required time.

TABLE.

FROM ANY DAY OF	TO THE SAME DAY OF											
	Jan	Feb.	Mar.	Apr	May	June	July	Aug	Sep.	Oct.	Nov.	Dec.
January	365	31	59	90	120	151	181	212	243	273	304	334
February	334	365	28	59	89	120	150	181	212	242	273	303
March	306	337	365	31	61	92	122	153	184	214	245	275
April	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	31	61	92	123	153	184	214
June	214	245	273	304	334	365	30	61	92	122	153	183
July	184	215	243	274	304	335	365	31	62	92	123	153
August	153	184	212	243	273	304	334	365	31	61	92	122
September	122	153	181	212	242	273	303	334	365	30	61	91
October	92	123	151	182	212	243	273	304	335	365	31	61
November	61	92	120	151	181	212	242	273	304	334	365	30
December	31	62	90	121	151	182	212	243	274	304	335	365

As an example, suppose that we want to know the time from the 15th February to the 15th of October. In the column of months on the left-hand side of the page, we find February, and right opposite, and under October, we find 242, the number of days required.

CIRCULAR MEASURE.

69. This measure is used in estimating angular magnitudes, and in reckoning latitude and longitude, and the motions of the heavenly bodies.

Every circle is divided into 360 equal parts called degrees, and these *are subdivided* into minutes and seconds.

TABLE.

60 seconds (″),	make	1 minute, marked	′.
60 minutes	"	1 degree, "	°.
30 degrees	"	1 sign, "	*s.*
12 signs, or 360°		1 circle. "	*cr.*

MISCELLANEOUS TABLE.

12 units	make	1 dozen.
12 dozen	"	1 gross.
12 gross	"	1 great gross.
20 units	"	1 score.
196 pounds	"	1 barrel of flour.
200 pounds	"	1 barrel of pork.
60 pounds	"	1 bushel of wheat.
30 pounds	"	1 bushel of oats.
56 pounds	"	1 bushel of corn, or rye.
70 pounds	"	1 bushel of salt.
24 sheets	"	1 quire.
20 quires	"	1 ream.

BOOKS.

A sheet folded in 2 leaves is named a *folio.*
" " " 4 " " a *quarto*, or 4*to.*
" " " 8 " " an *octavo*, or 8*vo.*
" " " 12 " " a *duodecimo*, or 12*mo.*

ADDITION OF DENOMINATE NUMBERS.

70. Suppose that we want to find the sum of £75 17*s.* 8*d.* 3 *far.*, and £53 18*s.* 2*d.* 2 *far.* We arrange the two denom-

inate numbers under each other, as represented in the following

OPERATION.

£	*s.*	*d.*	*far.*
75	17	8	3
53	18	2	2
£129	15	11	1

We then say that the sum of 2 far. and 3 far. is 5 far., which are equal to 1 penny and 1 far. The 1 far. we set down under the column of farthings, and add the 1 penny to the 2d. in the next column, and to this sum add the 8d., and obtain 11d. As 11d. is less than a shilling, we write 11d. under the column of pence, and pass to the column of shillings. The sum of 18s. and 17s. is 35 shillings, which are equal to 1 pound and 15 shillings, since 20, the number of shillings in a pound, is contained once in 35, with a remainder of 15. We set the 15s. under the column of shillings, and add the 1 pound to 53 pounds, and to this sum add the 75 pounds, and obtain £129. Hence the sum required is £129 15s. 11d. 1 far.

From the above process, we may derive, for the addition of Denominate numbers, the following

RULE.

I. *Arrange the numbers under each other, so that those of the same denomination may be in the same vertical column, and draw a line under them.*

II. *Find the sum of the numbers which are of the lowest denomination, as in simple addition, and divide this sum by the number of units of this denomination, which make one of*

*the next higher denomination.** *Set down the remainder under the numbers added, and add the quotient to the sum of the numbers of the next higher denomination.*

III. *Proceed in this manner through all the denominations, till the numbers of the highest denomination are reached, whose entire sum set down.*

EXAMPLES.

1. What is the sum of £45 19s. 4d., £125 18s. 9d. 3 far., £75 15s. 7d., 19s. 4d. 3 far., £325 11s. 8d. 1 far., £143 15s. 6d., 4d. 3 far., and £45 16s.? *Ans.* £763 16s. 8d. 2 far.

2. What is the sum of 8 yds. 3 qrs. 2 na., 25 yds. 3 qrs. 3 na., 17 yds. 2 na., 175 yds. 1 qr. 3 na. 2 in., 325 yds. 1 qr. 1 na. 1 in., 365 yds. 3 na. 2 in., and 45 yds. 3 qrs.
Ans. 963 yds. 3 qrs. 0 na. ½ in.

3. What is the sum of 18 mi. 3 fur. 21 rd. 11 yds. 2 ft., 8 fur. 16 rd. 10 yds. 1 ft., 65 mi. 7 fur. 17 rd. 7 in., 82 mi. 17 fur. 25 rd. 1 yd. 2 ft. 7 in., and 245 mi. 17 rd. 3 yds. 2 ft.? *Ans.* 414 mi. 5 fur. 20 rd. 5 yds. 2 ft. 2 in.

4. What is the sum of 2 cwt. 3 qrs. 15 lbs. 10 oz., 15 cwt. 1 qr. 10 lbs. 5 oz. 3 dr. 7 T. 16 cwt. 3 qrs. 19 lbs. 9 oz., and 27 T. 17 cwt. 1 qr. 23 lbs. 14 oz.?
Ans. 36 T. 12 cwt. 2 qrs. 19 lbs. 6 oz. 3 dr.

5. What is the sum of 17 lbs. 11 oz. 17 pwt. 17 gr., 25 lb. 10 oz. 14 gr., 225 lb. 7 oz. 14 pwt. 11 gr., 45 lb. 10 oz. 18 pwt. 15 gr., and 75 lb. 11 oz. 6 pwt. 8 gr.?
Ans. 391 lb. 3 oz. 23 pwt. 3 gr.

* NOTE.—When the sum of the numbers in any column is less than the number which makes one of the next higher denomination, set down this sum under the column added, and pass to the next column.

6. What is the sum of 15 A. 3 R. 17 P., 45 A. 2 R. 19 P., 3 R. 23 P., 263 A. 1 R. 17 P., 85 A. 0 R. 27 P., and 63 A. 3 R. 19 P.? *Ans.* 474 A. 3 R. 2 P.

7. What is the sum of 8 C. 3 c. ft., 25 C. 7 c. ft., 325 C. 6 c. ft., 65 C. 11 c. ft., and 365 C. 13 c. ft.? *Ans.* 793 C.

8. What is the sum of 3 wk. 5 da. 14 hr. 20 m. 15 sec., 17 da. 15 hr. 35 m. 45 sec., 1 wk. 4 da 17 hr. 42 m. 21 sec., 3 da. 37 hr. 18 m. 25 sec., and 3 wk. 6 da. 3 hr. 27 m. 33 sec.?
Ans. 12 wk. 3 da. 16 hr. 24 m. 19 sec.

9. What is the sum of 4 hhd. 30 gal. 3 pt., 25 hhd. 23 gal. 3 qt. 1 pt., 17 hhd. 18 gal. 0 qt. 2 pt., 14 hhd. 3 qt. 1 pt., 23 hhd. 17 gal. 3 qt. 1 pt., and 43 hhd. 18 gal.?
Ans. 127 hhd. 46 gal. 1 qt.

10. What is the sum of 8 lb. 11 ℥. 6 ʒ. 2 ℈., 9 lb. 10 ℥. 7 ʒ. 1 ℈., 4 lb. 7 ℥. 3 ʒ. 1 ℈., 17 lb. 8 ℥. 3 ʒ. 1 ℈., and 45 lb. 11 ℥. 3 ʒ. 1 ℈.? *Ans.* 87 lb. 2 ℥.

11. What is the sum of 10 rd. 3 yd. 1 ft. 7 in., 7 rd. 2 yd. 2 ft. 5 in., 3 rd. 4 yd. 1 ft. 9 in., 5 rd. 2 yd. 1 ft. 10 in., and 13 rd. 4 yd. 11 in.? *Ans.* 41 rd. ½ yd. 2 ft. 6 in.

12. What is the sum of 13° 10′ 25″, 14° 18′ 17″, 25° 35′ 45″, 27° 45′, 10° 16′ 45″, and 17′ 38″? *Ans.* 3 *s.* 1° 23′ 50″.

13. What is the sum of 145 bu. 3 pk. 1 qt., 163 bu. 1 pk. 3 qt., 275 bu. 2 pk. 7 qt., 45 bu. 3 pk. 6 qt., and 73 bu. 1 pk. 5 qt.? *Ans.* 704 bu. 0 pk. 6 qt.

14. What is the sum of 25 S. yd. 18 S. ft. 251 S. in., 65 S. yd. 18 S. ft. 284 S. in., 263 S. yd. 17 S. ft. 584 S. in., 47 S. yd. 21 S. ft. 54 S. in., and 65 S. yd. 17 S. ft. 965 S. in?
Ans. 468 S. yd. 11 S. ft. 410 S. in.*

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* NOTE.—The abbreviations S. ft., S. in., &c. stand for Solid ft., Solid in. *&c, and Sq. ft. Sq. in.* stand for Square feet, Square inches, &c.

15. What is the sum of 45 C. 23 Cu. ft. 25 Cu. in., 273 C. 75 Cu. ft. 684 Cu. in., 97 C. 18 Cu. ft. 384 Cu. in., 250 C. 64 Cu. ft. 197 Cu. in., and 264 C. 84 Cu. ft. 848 Cu. in.?

Ans. 931 C. 9 Cu. ft. 410 Cu. in.

16. What is the sum of 86 S. yd. 17 S. ft. 46 S. in., 245 S. yd. 18 S. ft. 289 S. in., and 265 S. yd. 17 S. ft. 1284 S. in.?

Ans. 597 S. yd. 25 S. ft. 1619 S. in.

SUBTRACTION OF DENOMINATE NUMBERS.

71. Suppose that we want to find the difference between £64 5s. 4d., and £43 17s. 7d. We arrange the two denominate numbers under each other as represented in the following

OPERATION.

£.	*s.*	*d.*
64	5	4
43	17	7
£20	7s.	9d.

We cannot substract 7d. from 4d. Therefore, we will add 12d. to 4d., and we have 16d., and then subtract 7d. from 16d. The remainder is 9d., which we place under the column of pence. Since we have increased the minuend by 12d.=1s., we must increase the subtrahend by an equal quantity. Hence, we add 1s. to 17s. in the subtrahend. Now, we cannot subtract 18s. from 5s., and, therefore, we add 20s. to 5s., and we have 25s., and then subtract 18s. from 25s., and we have 7s., which we set down under the column of shillings. Since we have increased the minuend by 20*s.*=£1, we must incease the subtrahend by an equal quantity. Hence, we add £1 to £43 in the subtrahend. By subtracting £44 from £64, *we have £20.* Hence, the difference required is £20 7s. 9d.

From the above process we derive, for the subtraction of denominate numbers, the following

RULE.

I. *Arrange the numbers as in addition of denominate numbers.*

II. *Commence at the lowest denomination, and subtract each number in the subtrahend from the corresponding one in the minuend, and set the remainder directly below.*

III. *If any number in the minuend is less than the corresponding one in the subtrahend, add as many units to this number as make one of the next higher denomination, and from this sum subtract the number in the subtrahend, and set the remainder below. Then add one to the number in the subtrahend of the next higher denomination, and proceed as before.*

EXAMPLES.

1. From the sum of £18 17s. 4d. and £25 19s. 9d., subtract the sum of 17s. 8d. and £12 4s. 11d. *Ans.* £31 14s. 6d.

2. From 17 yd. 3 qr. 3 na., subtract 11 yd. 1 qr. 2 na. *Ans.* 6 yd. 2 qr. 1 na.

3. From 49 A. 1 R. 15 P., subtract 24 A. 1 R. 18 P. *Ans.* 24 A. 3 R. 37 P.

4. From 23 mi. 7 fur. 17 rd. 4 yd., subtract 14 mi. 4 fur. 25 rd. 5 yd. *Ans.* 9 m. 2 fur. 31 rd. $4\frac{1}{2}$ yd.

5. If a person purchase 17 cwt. 1 qr. 17 lb. of sugar, and then sell 13 cwt. 3 qs. 19 lb., how much sugar will he have left? *Ans.* 3 cwt. 1 qr. 23 lb.

6. A merchant bought a piece of cloth containing 31 yd.

1 qr. 3 na., and sold from it at one time, 12 yd. 3 qr. 2 na., and at another time, 14 yd. 0 qr. 3 na. How many yards remained unsold? *Ans.* 4 yd. 1 qr. 2 na.

7. A man owns three farms; the first of which contains 75 A. 3 R. 17 P.; the second, 27 A. 1 R. 19 P.; and the third, as much as the other two lacking 4 A. 3 R. 29 P. How many acres did the third contain?

Ans. 98 A. 1 R. 7 P.

8. The latitude of New Orleans is 29° 8′ 32″, and that of New York is 40° 42′ 35″. What is the difference of latitude of these two places? *Ans.* 11° 34′ 3″.

9. The longitude of Boston is 71° 4′ 20″, and that of Buffalo is 78° 55′ 0″. What is the difference of longitude of these two places? *Ans.* 7° 50′ 40″.

10. George Washington was born on the 22d day of February, 1732, and he died December 14, 1799. To what age did he live? *Ans.* 67 yr. 9 mo. 22 da.*

11. William Shakspeare was born on the 23d of April, 1564, and he died April 3, 1616. What was his age?

Ans. 51 yr. 11mo. 10 da.

12. Newspapers were first published in England, by order of Queen Elizabeth. One of these, entitled the English Mercury, dated July 28, 1588, is remaining in the British Museum. How many years have elapsed from the date of this paper to the present time? *Ans.*

13. From 10 bu. 3 pk. 1 qt. 1 pt. take 7 bu. 2 pk. 3 qt. 1 pt. *Ans.* 3 bu. 6 qt.

14. From 12 hhd. 34 gal. 2 qt. 1 pt. 3 gi. take 9 hhd. 45 gal. 3 qt. 1 pt. 2 gi. *Ans.* 2 hhd. 51 gal. 3 qt. 1 gi.

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* *Note.*—The month is estimated at 30 days.

MULTIPLICATION OF DENOMINATE NUMBERS.

72. Let it be required to multiply 7 yd. 3 qr. 3 na. by 8. We arrange the factors as represented in the following

OPERATION.

yd.	qr.	na.
7	3	3
		8
63	2	0

First, we say that 8 times 3 na. are 24 na. Since 4 na. make 1 quarter, 24 na., will make as many quarters as 4 is contained times in 24. 4 is contained 6 times in 24, with no remainder. Since there is no remainder, there can be no nails in the product, and we therefore set down a 0 in the place of nails. Then we say that 8 times 3 quarters are 24 quarters, and by adding the 6 quarters, we have 30 quarters, which are equal to 7 yards and 2 quarters. We set down the two quarters, and reserve the 7 yards to add to the next product. Finally, 8 times 7 yards are 56 yards, and by adding the 7 yards to this product, we obtain 63 yards. Hence, the product required is 63 yards, 2 quarters, 0 nails.

From the above process we derive, for the multiplication of Denominate Numbers, the following

RULE.

Multiply the number in the lowest denomination by the multiplier, and divide the product by the number which expresses how many units of this denomination make one of the next higher denomination. Set down the remainder of this division under the number multiplied, and add the quotient

to the product of the number in the next higher denomination and the multiplier, and then proceed as before.

If either one of the products is not equal to one of the next higher denomination, set down the entire product. Lastly, set down the entire product of the highest denomination.

EXAMPLES.

1. How much land is there in three lots, each of which contains 13 A. 3 R. 17 P.? *Ans.* 41 A. 2 R. 11 P.

2. What is the weight of 3 hogsheads of sugar, each of which weighs 9 cwt. 3 qr. 21 lb.? *Ans.* 1 T. 9 cwt. 3 qr. 13 lb.

3. How much cloth will it require to make 21 suits of clothes, if 1 suit requires 7 yd. 1 qr. 3 na.?* *Ans.* 156 yd. 0 qr. 3 na.

4. If a man can hoe an acre of corn in 10 hr. 25 min. 40 sec., in what time can he hoe 48 acres? *Ans.* 500 hr. 32 m.

5. How much wood can a team draw in 15 loads, if it can draw 1 C. 24 S. ft. at each load? *Ans.* 17 C. 104 S. ft.

6. What is the weight of 24 silver spoons, if each weighs 3 oz. 17 pwt. 16 gr.? *Ans.* 7 lb. 9 oz. 4 pwt.

7. What is the cost of 45 acres of land, at £17 8s. 10d. per acre? *Ans.* £784 17s. 6d.

8. If a man can walk 25 mi. 7 fur. 25 rd. in 1 day, how far can he walk in 16 days? *Ans.* 415 mi. 2 fur. 0 rd.

9. If a man can saw 1 cord of wood in 7 hr. 45 min. 50 sec., how long will it take him to saw 12 cords? *Ans.* 93 hr. 10 min.

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* *Note.—Multiply* the multiplicand by one of the factors of 21 *and that product by* the other.

10. The length of each side of a square field is 3 fur. 25 rd.; what is the whole distance around the field?

Ans. 1 mi. 6 fur. 20 rd.

11. What is the cost of 12 cwt. of sugar, at £3 7s. 4d. per cwt.? *Ans.* £40 8s.

12. If a family consume 12 bu. 8 pk. 6 qt. of flour in 1 month, what quantity will the same family consume in 12 months? *Ans.* 170 bu. 1 pk.

DIVISION OF DENOMINATE NUMBERS.

73. Let it be required to divide £345 6*s.* 8*d.* by 14. For this division we have the following

OPERATION.

```
      £   s.  d.      £   s.  d.
14)345   6   8     (24  13   4
   28
   --
    65
    56
    --
     9
    20
14)186 (13s.
   14
   --
    46
    42
    --
     4
    12
 14)56 (4d.
    56
    --
     0
```

First, by the rule for simple division, we find that 14 is contained 24 times in £345, and the remainder is £9. Since 20s. make £1, £9=9×20s.=180s., to which we add the 6s. in the dividend, and obtain 186s. We now divide 186s. by 14. 14 is contained in 186, 13 times, and the remainder is 4s. Since 1s.=12d., 4s.=4×12d.=48d., to which we add the 8d. in the dividend, and obtain 56d., and 14 is contained 4 times in 56. Hence, the $\frac{1}{14}$ part of the dividend, or the quotient required, is £24 13s. 4d.

Hence, for the division of a denominate number by any whole number, we have the following

RULE.

Divide the number of the highest denomination by the divisor, and reduce the remainder, if any, to the next lower denomination, and to this result, add the number in the dividend of the same denomination. Divide this sum by the divisor, and proceed in this manner through all the denominations of the dividend.

EXAMPLES.

1. Divide £23 15s. 7½d. by 37. *Ans.* £0 12s. 10d. 1 far.
2. Divide £199 3s. 10d. by 53. *Ans.* £3 15s. 2d.
3. Divide £675 12s. 6d. by 138. *Ans.* £4 17s. 11d.
4. Divide £315 3s. 10d. 1 far. by 365. *Ans.* £0 17s. 3d. 1 far.
5. Divide 23 lb. 7 oz. 6 pwt. 12 gr. by 7.* *Ans.* 3lb. 4oz. 9pwt. 12gr.

**Note.*—When the divisor is less than 12, employ short division;

6. Divide 1061 cwt. 2 qr. 26 lb. by 28. *Ans.* 37 cwt. 3qr. 17lb.

7. Divide 375 mi. 2 fur. 7 rd. 2 yd. 1 ft. 2 in. by 39.

Ans. 9 mi. 4 fur. 39 rd. 0 yd. 2ft. $6\frac{32}{39}$ in.

8. Divide 51 A. 1 R. 11 P. by 51. *Ans.* 1 A. 0 R. 1 P.

9. Divide 571 yd. 2 qr. 1 na. by 47.

Ans. 12 yd. 0 qr. $2\frac{27}{47}$ na.

10. Divide 120 mo. 2 wk. 3 da. 4 hr. 24 min. by 111.

Ans. 1 mo. 0 wk. 4 da. 3 hr. 4 min.

11. A merchant bought 12 pieces of cloth, each piece containing 31 yd. 3 qr., and then sold $\frac{3}{4}$ of what he had purchased. What quantity of cloth had he left? *Ans.* 95 yd. 1 qr.

12. If a man can walk 1011 miles in 40 days, how far can he walk in 1 day? *Ans.* 25 mi. 2 fur. 8 rd.

13. If 106 tons of iron cost £2002 19s. 4d., what is the cost of 1 ton? *Ans.* £18 17s. $11\frac{1}{53}$d.

14. If 27 sticks of timber of the same size, measure 487 Cu. ft. 864 Cu. in., what will 1 stick measure?

Ans. 18 Cu. ft. 96 Cu. in.

REDUCTION OF DENOMINATE NUMBERS.

The process of reducing a denominate number, or quantity, from a higher to a lower denomination, is called *Reduction Descending*, and that of reducing a denominate number, or quantity, from a lower to a higher denomination, is called *Reduction Ascending.*

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and if the divisor is a composite number, divide the dividend by one of its factors, and the quotient by another factor, and so on till all the factors have been used.

REDUCTION DESCENDING.

74. For performing the operations in Reduction Descending, we have the following

RULE.

I. *Multiply that term of the denominate quantity, which is of the highest denomination, by a number which expresses how many units of the next lower denomination make one of the highest denomination, and to the product add the term in the given quantity which is of the same denomination as the product. In like manner, reduce this result to the next lower denomination, and so on, till the required reduction has been made.*

II. *If the denominate quantity be a fraction, the integers in the successive products will form the denominate quantity required.*

EXAMPLES.

1. Reduce £17 8s. 4d. to pence.

OPERATION.

£	*s.*	*d.*
17	8	4

20
340s.
8s.
348s.
12
4176d.
4d.
4180d.

Since 20s.=£1, £17=17×20s.=340. 340s.+8s.=348s. Since 12d.=1s., 348s.=348×12d.=4176d., and 4176d.+4d. =4180d. Hence £17 8s. 4d.=4180d.

2. Reduce £$\frac{5}{6}$ to shillings and pence.

OPERATION.

$$\frac{5}{6}\times\frac{20}{1}=\frac{5\times\overset{10}{\cancel{20}}}{\cancel{6}3}=\frac{50}{3}=16\tfrac{2}{3}\text{s.}$$

$$\overset{\text{s.}}{\frac{2}{3}}\times\frac{12}{1}=\frac{2\times\overset{4}{\cancel{12}}}{\cancel{3}}=8\text{d.}$$

Since £1=20s., £$\frac{5}{6}$=$\frac{5}{6}$ of 20s.=16$\frac{2}{3}$s. Again, since, 1s.= 12d., $\frac{2}{3}$s.=$\frac{2}{3}$ of 12d.=8d.

3. Reduce £0.345 to lower denominations.

OPERATION.

$$\begin{array}{r}
£0.345\\
20\\
\hline
6.900\text{s.}\\
12\\
\hline
10.800\text{d.}\\
4\\
\hline
3.200\text{ far.}
\end{array}$$

Hence, £0.345=6s. 10. 3.2 far. The pupil may give the explanation.

4. What is the value of $\frac{2}{3}$ of an acre?

Ans. 2 R. 26 P. 20 Sq. yd. 1 Sq. ft. 72 Sq. in.

5. Reduce 5 A. 3 R. 27 P. to poles. *Ans.* 947 P.

6. Reduce £17 18s. 7d. 4 far. to farthings.

Ans. 17216 far.

7. What is the value of 0.13456 of a mile?

Ans. 1 fur. 3 rd. 0 ft. 11.7 in.

8. Reduce £24 17s. to farthings. *Ans.* 21936 far.

9. What is the value of 0.835 A. in roods, rods, and yards?

Ans. 3 R. 13 rd. $18\frac{3}{20}$ Sq. yd.

10. Reduce 1 R. 24 P. to square feet. *Ans.* 17424 Sq. ft.

11. Reduce 0.475 of a degree to minutes and seconds.

Ans. 28′. 30″.

12. Reduce $\frac{7}{30}$ of a week to days, hours, and minutes.

Ans. 1 da. 15 h. 12 m.

13. Find the value of $\frac{7}{12}$ of a cubic yard, in cubic feet and inches. *Ans.* 15 Cu. ft. 1296 Cu. in.

14. Reduce 15 bu. 2 pk. 5 qt. to quarts. *Ans.* 501 quarts.

15. Find the value of $\frac{3}{12}$ of a quarter in pounds, ounces, and drams. *Ans.* 16 lbs. 10 oz. $10\frac{2}{3}$ dr.

16. Reduce 17 yd. 3 qr. 3 na. to nails. *Ans.* 287 na.

17. Reduce 1000 barrels of molasses to gallons, and find their cost at $37\frac{1}{2}$ cents per gallon. *Ans.* \$11812.50.

18. What is the cost of 45.385 acres of land, at the rate of 35 dollars for 60 square rods? *Ans.* \$4235.$93\frac{1}{3}$.

19. Find the value of $\frac{7}{12}$ of an ounce, apothecaries' weight.

Ans. 4 ʒ 2 ℈.

20. Reduce 12 cu. yd. 17 cu. ft. to cubic inches.

Ans. 589248 Cu. in.

21. Reduce 142 yd, 3 qr. 2 na. to nails. *Ans.* 2286 na.

22. What is the value of 0.345 of a barrel of flour?

Ans. 67 lb. 9 oz. $14\frac{1}{2}\frac{8}{5}$ dr.

23. What is the value of 0.275 of a bushel of wheat?

Ans. 16 lb. 8 oz.

24. Find the value of 0.34214 of a year of 365 days, in lower denominate numbers. *Ans.* 124 da. 21 h. 8 m. $47\frac{1}{25}$ sec.

REDUCTION ASCENDING.

75. For performing the operations in Reduction Ascending, we have the following

RULE.

Divide that term of the given quantity, which is of the lowest denomination, by the number which expresses how many of this denomination make one of the next higher denomination; the quotient will be of that higher denomination. Add this quotient to the term in the given quantity, which is of the same denomination as the quotient, (if there be any such term,) and then reduce this result to the next higher denomination, and so on, till the required reduction has been made.

EXAMPLES.

1. Reduce 6 fur. 30 rd. 12 ft. $8\frac{4}{13}$ in. to the fraction of a mile.

OPERATION.

$$12 \;\Big|\; 8\frac{4}{13}\text{ in.} \quad \overset{\text{in.}}{\frac{108}{13}} \div 12 = \frac{\overset{9}{\cancel{108}}}{13} \times \frac{1}{12} = \frac{9}{13}\text{ ft.}$$

$$16\tfrac{1}{2} \;\Big|\; 12\text{ ft.} + \frac{9}{13}\text{ft.} = 12\frac{9}{13} = \frac{165}{13}. \quad \frac{165}{13} \div 16\tfrac{1}{2} = \frac{10}{13}\text{ rd.}$$

$$40 \;\Big|\; 30\text{ rd.} + \frac{10}{13}\text{ rd.} = \frac{400}{13}. \quad \frac{400}{13} \div 40 = \frac{\overset{10}{\cancel{400}}}{13} \times \frac{1}{\cancel{40}} = \frac{10}{13}\text{ fur.}$$

$$8 \;\Big|\; 6\text{ fur.} + \frac{10}{13}\text{ fur.} = \frac{88}{13}. \quad \frac{88}{13} \div 8 = \frac{11}{13}\text{ mi.}$$

$\frac{11}{13}$ mi. *Ans*

Since 12 inches make 1 foot, we divide $8\frac{4}{13}$ inches by 12, in order to reduce them to feet. We find that $8\frac{4}{13}$ in. $=\frac{9}{13}$ ft. We now add the $\frac{9}{13}$ of a foot to the 12 feet, and obtain $12\frac{9}{13}$ feet $=\frac{165}{13}$ of a foot. In order to reduce $\frac{165}{13}$ of a foot to the fraction of a rod, we divide $\frac{165}{13}$ by $16\frac{1}{2}$, or $\frac{33}{2}$, since $16\frac{1}{2}$ feet make 1 rod. We find that $\frac{165}{13}$ ft. $=\frac{10}{13}$ rd. This $\frac{10}{13}$ rd. we add to the 30 rd., and obtain $30\frac{10}{13}$ rods, or $\frac{400}{13}$ of a rod. Since 40 rods make 1 fur., we divide $\frac{400}{13}$ rd. by 40, and find that $\frac{400}{13}$ rd. $=\frac{10}{13}$ fur. This $\frac{10}{13}$ of a furlong we add to the 6 furlongs, and obtain $6\frac{10}{13}$ furlongs $=\frac{88}{13}$ of a furlong, which we divide by 8, since 8 furlongs make 1 mile. We find that $\frac{88}{13}$ fur. $=\frac{11}{13}$ mi., which is the fraction required.*

2. Reduce 17s. 9d. to the decimal of a pound.

OPERATION.

12	9d.
20	17.75s.
	0.8875 of a £.

Here, we find that 9d. is equal to 0.75 of a shilling, which we add to 17s. and obtain 17.75s. Dividing 17.75s. by 20, since 20s. make £1, we obtain for the quotient £0.8875.

3. Reduce 875 pence to pounds, shillings, and pence.

OPERATION.

12	875d.	
20	72s.	11d. rem.
	£3	12s. rem.

* Note.—This result might be obtained by reducing the given quantity to *thirteenths of an inch*, for the numerator, and 1 mile *to thirteenths of an inch* for the denominator of a fraction, and

£3 12s. 11d.

Since 12d. make 1s., there are as many shillings in 875d. as 12 is contained times in 875. 12 is contained 72 times in 875, with a remainder of 11; hence 875d.=72s. 11d. Since 20s. make £1, 72s. are equal to as many pounds as 20 is contained times in 72. 20 is contained 3 times in 72, with a remainder of 12s. Hence, 875d.=£3 12s. 11d.

3. Reduce 48 rods to the fraction of an acre.

OPERATION.

$$\frac{\overset{\overset{3}{\cancel{12}}}{\cancel{48}}}{1}\times\frac{1}{\underset{10}{\cancel{40}}}\times\frac{1}{\cancel{4}}=\frac{3}{5}$$

Since 40 rods make 1 rood, we divide 48 by 40, or multiply it by $\frac{1}{40}$; and since 4 roods make 1 acre, we divide this quotient by 4, and thus find that 48 rods=$\frac{3}{5}$ of an acre.

4. Reduce 4s. 8d. 3 far. to the decimal of a pound.

Ans. 0.23645+.

5. Reduce 3 R. 27 P. to the decimal of an acre, and find the cost of this quantitity of land at $75 per acre.

Ans. A .91875; cost $68.90625.

6. Reduce 17 cwt. 3 qr. 21 lb. to the decimal of a ton, and find the cost of this quantity of hay at $6.75 per ton.

Ans.

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then reducing the fraction to its lowest terms. As an example, reduce 8s. 4d. to the fraction of a pound. 4s. 4d.=52d. £1=240d. Hence, 1d.=$\frac{1}{240}$ of a pound, and 52d.=52 times £$\frac{1}{240}$=£$\frac{52}{240}$=£$\frac{13}{60}$.

7. Reduce 3 qr. 3 na. to the decimal of a yard, and find the cost of this quantity of cloth at $7.25 per yard.

Ans. Cost $6.796875.

8. Reduce 13 ounces to the decimal of a pound, and then find the cost of 4 lb. 13 oz. of tea, at 75 cents per pound.

Ans. $3.62, nearly.*

9. Reduce 14 cwt. 3 qr. 16 lb. to the fraction of a ton, and then find the cost of this quantity of iron at $50 per ton.

Ans. $37.275.

10. Reduce 35 rd. 9 ft. 2 in. to the fraction of a furlong.

Ans. $\frac{8}{9}$.

11. Reduce 10 oz. 13 pwt. 8 gr. to the fraction of a pound.

Ans. $\frac{8}{9}$.

12. Reduce 2 qr. 0 na. $1\frac{5}{13}$ in. to the fraction of a yard.

Ans. $\frac{7}{13}$.

13. Reduce 7 hr. 18 min. 24 sec. to the decimal of a day.

Ans. 0.3044 +.

14. Reduce 475 qr. to yards and quarters.

Ans. 118 yd. 3 qr.

15. Reduce $\frac{1}{2}\times\frac{3}{10}$ of $4\frac{2}{3}$ in. to the fraction of a yard.

Ans. $\frac{7}{360}$.

16. Find the sum of 14 A. 3 R. 15 P. and 3 R. 35 P. in acres and the decimal of an acre. *Ans.* 15.7125 acres.

17. What is the cost of 75 A. 3 R. 17 P., at $45 per acre.

Ans. 3413.53\frac{1}{8}$.

18. What is the cost of 36 bu. 24 lb. of wheat at 87 cents per bushel? *Ans.* $31.688.

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* *Verify* this result by giving different solutions.

19. What is the cost of 48 bu. 28 lb. of barley at 56 cents per bushel? *Ans.* \$27.22 nearly.

20. What is the cost of 74 bu. 15 lb. of oats at 28 cents per bushel? *Ans.* \$20.86.

21. What is the cost of 384 pounds of flour, at \$5.75 per barrel? *Ans.* \$11.36.

22. What is the cost of 54 lb. 7 oz. of butter at 18 cents per pound? *Ans.* \$9.79875.

23. Find the sum of 7 A. 1 R. 16 P., 3 R. 14 P., and 2 R. 32 P., in acres. *Ans.* 8.8875 acres.

24. Find the sum of $\frac{3}{8}$ of a rood, and $\frac{5}{12}$ of a rod, in the decimal of an acre. *Ans.* 0.09635 acres.

CHAPTER IX.

PERCENTAGE AND ITS APPLICATIONS.

PERCENTAGE.

76. Percentage is an allowance made by the *hundred* on any number, or quantity. The term *per centum* signifies by the hundred. The abbreviation, *per cent.*, is generally used for *per centum.* The expression 15 per cent. of 18 dollars, means fifteen hundredths of 18 dollars. We may also regard it as meaning an allowance of fifteen hundredths of a dollar on each dollar. It is obvious that an allowance of fifteen hundredths on each dollar, is equivalent to an allowance of fifteen dollars on one hundred dollars. The allowance made *per centum* is called the *rate per cent.*

From the definition of the term *per cent.*, it follows that the *rate per cent.* may be expressed by a decimal fraction. Thus, 7 per cent. may be expressed 0.07 ; $4\frac{1}{2}$ per cent. $0.04\frac{1}{2}$, or 0.045 ; 108 per cent. 1.08 ; $\frac{7}{8}$ per cent. $0.00\frac{7}{8}$, or 0.00875.

To determine the percentage on any number or quantity, it is plain that we may use the following

RULE.

Multiply the number by the decimal fraction which expresses the rate per cent.

EXAMPLES.

1. What is 7 per cent. of $2312 ?

OPERATION.

$$\begin{array}{r} \$2312 \\ 0.07 \\ \hline \$161.84 \end{array}$$

Since the percentage on 1 dollar is $0.07, on $2312 it is 2312 times $0.07 or $161.84.

2. What is $6\frac{2}{3}$ per cent. of $3485 ? *Ans.* 23.23\frac{1}{3}$.

3. What is 25 per cent. of $484 ? *Ans.* $121.

4. What is 18 per cent. of $384604 ? *Ans.* $69228.72.

5. A farmer raised 3840 bushels of wheat, and gave 9 per cent. of it for thrashing, and 8 per cent. of the remainder for flouring. How many bushels remained ?

Ans. 3214.848 bushels.

6. A merchant deposited $4540 in a bank, and drew out at one time 23 per cent. of it, and at another time 27 per cent. of it. How many dollars did he have remaining in the bank?

Ans. $2270.

7. If a man's salary is $850 per year, and he lays up $37\frac{1}{2}$ per cent. of it, how much does he save per year ?

Ans. $531.25

8. What is 8 per cent. of £145 16s. 8d. ?*

Ans. £11 13s. 4d.

9. What is 10 per cent. of £14 3s. 4d. ?

Ans. £1 8s. 3.8d.

* NOTE.—Reduce the 16s. 7d. to the decimal of a pound.

10. What is 12 per cent. of 20 T. 14 cwt. 3 qrs. 16 lb.?

Ans.

11. What is 15 per cent. of 62 yd. 3 qr. 2 na.?

Ans. 9 yd. 1 qr. 28 na.

12. A man purchased a house and lot for $2500, and sold them so as to lose 17 per cent. He then used the money that he received for his house and lot in a speculation, and gained $33\frac{1}{2}$ per cent of it. How much did his gain exceed his loss?

Ans. 266.66\frac{2}{3}$.

13. A merchant purchased iron at $4.74 per hundred weight, and paid 75 cents per hundred weight for freight. How must he sell his iron per hundred weight, and gain 25 per cent.?

Ans. 6.86\frac{1}{4}$.

14. A merchant purchased lumber at $8.75 per thousand; how must he sell it by the hundred, and gain 25 per cent.?

Ans. 1.093\frac{3}{4}$.

INSURANCE, COMMISSION, BROKERAGE, STOCKS.

DEFINITIONS.

1. *Insurance* is a contract by which one party agrees to pay to another, for a stipulated consideration, a specified sum in case the property insured is destroyed or injured by any means mentioned in the contract. The written agreement, or contract, is called the *policy*, and the sum paid for the insurance is called the *premium*, which is estimated at a certain per cent. on the sum insured.

2. Life Insurance is a contract for the payment of a certain sum on the death of the person whose life is insured.

3. *Commission* is an allowance made to a factor or commission merchant for selling and buying goods. It is estimated at a certain rate per cent. on the value of the goods bought or sold.

4. *Brokerage* is the amount charged by dealers in money or stocks, who are called brokers, for purchasing stocks, and transacting other business for their employers. Brokerage is estimated in the same way as commission.

5. Stocks are Government Bonds and the capital of moneyed institutions, such as banks, railroad companies, &c.

6. Stocks are generally divided into shares of $100 each. The owners of these shares are called Stockholders. The *par value* of a share is what it originally cost. When a share will bring more in market than its original cost, it is said to be *above par;* and when it will sell for less than its original cost, it is said to be *below par.* The stock then sells at a discount.

EXAMPLES.

1. What will be the annual premium for the insurance of a house valued at $2500, at $\frac{3}{8}$ per cent.? *Ans.* $9.375.

2. What will be the insurance on a shipment of goods, valued at $6780, from Boston to New Orleans, at $1\frac{3}{4}$ per cent.? *Ans.* $118.65.

3. What will it cost to insure a ship and her cargo from New-York to London, the ship being valued at $25000, and the cargo at $18250, at $2\frac{1}{4}$ per cent.? *Ans.* $973.125.

4. A steamboat valued at $24000 is insured at $2\frac{3}{4}$ cents per annum. What is the annual premium? *Ans.* $660.

5. A gentleman at the age of 25 effects an insurance of

\$2500 for life, at the rate of \$1.94 on \$100 per annum. What was his annual premium? *Ans.* \$48.50.

6. A person at the age of 25 effects an insurance of \$3840, for the term of 7 years, at the rate of \$1.07 on \$100 per annum. What is his annual premium? *Ans.* \$41.088.

7. If a person at the age of 28 should effect an insurance of \$3000 for life at the rate of \$2.16 on 100 per annum, and he should die at the age of 70, how much would the insurance exceed the sum of the annual premiums? *Ans.* \$278.40.

8. A person at the age of 20 effects an insurance of \$4500 on his life for the term of 3 years, at the rate of \$0.86 on \$100. What was his annual premium? *Ans.* \$38.70

9. What is the commission for selling \$9000 worth of goods, at 2 per cent.? *Ans.* 180

10. A lady having \$10480, paid an agent $1\frac{1}{4}$ per cent. commission per annum, for taking care of it. What did his annual commission amount to? *Ans.* \$131.

11. A gentleman paid a broker $\frac{1}{4}$ per cent. for purchasing railroad stock to the amount of \$30500. What did the brokerage amount to? *Ans.* \$76.25.

12. A merchant who failed found that he could pay 37 per cent. of his debts. How much can he pay on a debt of \$480? *Ans.* 177.60.

13. A commission merchant in New York received 100 dozen of eggs, 1480 pounds of ham, 1580 pounds of butter, and a 1000 pounds of cheese, to be sold on commission. If his commission is $3\frac{1}{2}$ per cent., and he sells the eggs at $18\frac{3}{4}$ *cents per dozen,* the ham at 15 cents per pound, the butter at

23 cents per pound, and the cheese at $8\frac{3}{4}$ cents per pound, what will his commission amount to? *Ans.* \$24.21475.

14. The par value of 380 shares of bank stock is \$100 per share. What is the present value of these stocks, if the stock is 15 per cent. below par? *Ans.* \$32300.

15. The par value of 250 shares of railroad stock is \$100 per share. What is the present value of these shares, if the stock is 18 per cent. above par? *Ans.* \$29500.

16. A bank fails, and has bills in circulation to the amount of \$284657. It can pay only 15 per cent. of this amount. How much money has it on hand? *Ans.* \$42698.55.

17. I direct my broker to purchase 25 shares of the Michigan Central Railroad stock, the par value being \$100 per share, and the stock is 15 per cent. above par. What will the 25 shares cost me, if the broker charges $\frac{1}{4}$ per cent.? *Ans.* \$2882.18$\frac{3}{4}$.

18. I directed a broker to purchase 12 shares of the New-York and Erie Railroad stocks, the par value being \$100 per share. The stock sells at a discount of 22 per cent., and the broker charges me $\frac{1}{4}$ per cent. What will the 12 shares cost?

Ans. \$938.34.

19. I send my broker \$6324, with which I direct him to purchase stocks, after taking out his commission of 2 per cent. How much money will he expend in purchasing stocks?*

Ans. 6200.

20. I sold 2380 bushels of wheat at $87\frac{1}{2}$ cents per bushel,

* NOTE.—By the question, the broker is to receive 2 cents for every dollar's worth of stock which he purchases. Hence, if I had sent him \$1.02, he could have purchased a dollar's worth of stock, and reserved the 2 cents for his commission. Hence, as often as he receives \$1.02, or as many times as \$1.02 is contained times in \$6324, so many dollars worth *of stock* he can purchase.

for a wheat buyer, at a commission of 10 per cent. How much money ought I to pay over to my principal?

Ans. \$1874.25.

21. I sent to my agent \$2450, and directed him to purchase iron with it. If I allow him $2\frac{1}{2}$ per cent. commission on the money which he expends, what will be the value of the iron which he purchases? *Ans.* \$2511.25.

22. How many shares of bank stock can be purchased for \$1350, the par value of a share being \$100, and the stock selling at a discount of 25 per cent. *Ans.* 18 shares.

23. How much stock which is $7\frac{1}{2}$ per cent. above par can be purchased for 4741, $\frac{1}{4}$ per cent. on the par value of the stock being paid for brokerage? *Ans.* \$4400.

24. A speculator directed his broker to purchase 3 shares of the Michigan Southern Railroad stock at \$100 per share. If the stock is 25 per cent. above par, and he pays $\frac{1}{4}$ per cent. for brokerage, what will his stock cost him?

Ans. \$379.68$\frac{3}{4}$.

25. What amount of stock in the capital of a Cotton Factory can be purchased for \$1743, the stock being sold at a discount of 17 per cent.? *Ans.* \$2100.

26. A merchant ships from New-York to London goods valued at \$12000. He wishes to insure for an amount which is equal to the value of the goods increased by the premium for insurance. For how much must he insure, at $2\frac{1}{2}$ per cent.?

Ans. \$12300.

27. The owner of a house, which cost \$3000, wishes to insure it for such a sum that, in case it is destroyed by fire, he may obtain from the insurance company the value of his house and the amount paid for insurance. For what sum must he insure it, the rate of insurance being $\frac{3}{4}$ per cent.?

Ans. \$3022.67.

DUTIES.

DEFINITIONS.

1. Duties are taxes imposed by the government on imported goods. Importers of goods are required by law to land them at particular places, called ports of entry, and the duties are paid at *Custom Houses* established in all ports of entry.

2. Duties are of two kinds, *specific* and *advalorem*. *A specific duty* is one that is imposed upon *quantity*, without regard to its value; as a certain sum upon a ton of iron. An *advalorem duty* is a certain percentage on the estimated *value* of the goods.

3. *Gross weight* is the weight of the goods, together with that which contains them.

4. *Tare* is an allowance which is made for the weight of that which contains the goods; as the box, bag, cask, &c.

5. *Draft* is an allowance made on the gross weight for waste. It is deducted before other allowances are made.

6. *Leakage* is an allowance for the waste of liquors contained in casks. The allowance is generally two per cent. on liquors, on which a duty is paid per gallon.

The usual allowance for *Draft* is given in the following

TABLE.

					Allowance.
On 112 lbs.					1 lb.
From 112	"		to	224 lbs.	2 lbs.
" 224	"		"	336 "	3 lbs.
" 336	"		"	1120 "	4 lbs.
" 1120	"		"	2060 "	7 lbs.
" 2060	"	and upwards.			9 lbs.

77. The rules and principles already given, are sufficient for solving the examples under the head of Duties. All allowances must be made before the duty is computed.

EXAMPLES.

1. What is the duty on 10 boxes of chocolate weighing 240 lbs. each, at 3 cents per pound, the draft being allowed as in the table, and the tare being 10 per cent?

OPERATION.

Gross weight, 2400 lbs.
Deduct draft, 9 lbs.
2391 lbs.
10 per cent. of 2391 lbs, = 239.1 "
Net weight, 2051.9 lbs.
$\$0.03 \times 2051.9 = \$615.57$, duty.

2. What is the duty on 196 tons of iron at $22.75 per ton? *Ans.* $4459.

3. What is the duty on 8 hhds. of wine, at $8\frac{3}{4}$ cents per gallon, the leakage being 2 per cent.? *Ans.* $43.218.

4. What is the duty on 380 bags of coffee, the gross weight of each bag being 175 lbs., invoiced * at 6 cents per pound; the tare being 5 per cent., and the duty 18 per cent.? *Ans.* $678.39.

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* NOTE.—An invoice is a full description of goods, in which the price of the merchandize is stated.

When the gross weight is given, and the draft is not mentioned, *deduct the draft as* given in the table.

5. What is the advalorem duty, at 25 per cent., on a quantity of cloths, invoiced at \$3450? *Ans.* \$862.50.

6. What is the duty on 8 casks of Glauber salts, each weighing 175 lbs. gross, the tare being 8 per cent., and the duty 3 cents on a pound? *Ans.* \$38.42.

7. What is the duty on 21 bags of coffee, each weighing 120 lbs. gross, invoiced at 7 cents per pound; the duty being 15 per cent., and the tare 2 per cent.? *Ans.* \$25.71.

8. What is the ad valorem duty on an invoice of silk goods, which cost at Canton \$3840, at 75 per cent.? *Ans.* \$2880.

9. What is the duty, at 13 cents per gallon, on 24 casks of wine, each containing 60 gallons, an allowance of 2 per cent. being made for leakage? *Ans.* \$183.456.

10. What is the duty, at 10 cents per pound, on 45 chests of tea, each weighing 125 lbs., the tare being 12 lbs. per chest? *Ans.* \$508.50.

11. The duty on a box of goods, at 37½ per cent., was \$72; what is the amount for which these goods were invoiced? *Ans.* \$192.

12. A merchant purchased cutlery in Sheffield, England, for which his bill was \$2384.62½. What was the duty on this amount, at 8¾ per cent.? *Ans.* \$208.554.

13. A merchant imported 240 bags of coffee, weighing gross 116 lbs. each, the tare being 2 per cent., and the duty 3½ cents per pound. What amount of duty did he pay? *Ans.* \$946.68.

14. A merchant purchases goods, invoiced at \$3845, and pays a duty on them of 25 per cent. Freight and other charges on the goods amount to \$384.45. For what sum must he sell these goods, and make 33⅓ per cent.? *Ans.* \$6920.93⅓.

15. The duty on a bale of Irish linens was \$360, at $18\frac{3}{4}$ per cent. What was the amount invoiced? *Ans.* \$1920.

16. The advalorem duty at $33\frac{1}{3}$ per cent., on a box of books was \$390. For how much were the books invoiced?
Ans. \$1170.

17. What is the duty on a lot of bombazines, invoiced at \$480, at $18\frac{3}{4}$ per cent.? *Ans.* \$90.

18. A merchant imported 4860 pounds o tea, which cost him $33\frac{1}{3}$ cents per pound, and he paid an advalorem duty of $12\frac{1}{2}$ per cent., and other charges amounting to \$450. How must he sell his tea per pound in order to gain 25 per cent.?
Ans. \$0.47, nearly.

19. A merchant purchased goods invoiced at \$4800, and he paid an advalorem duty of $8\frac{1}{3}$ per cent. For how much must he sell his goods in order that he may gain $16\frac{2}{3}$ per cent?
Ans. $\$6066\frac{2}{3}$.

20. A merchant imports 3200 yards of cotton cloth, invoiced at \$360, and he pays an advalorem duty of 10 per cent., and other charges amounting to \$45. How must he sell his cloth per yard in order that he may gain $33\frac{1}{3}$ per cent. on the money paid out? *Ans.* $\$0.18\frac{1}{2}$, nearly.

21. What is the duty, at $33\frac{1}{3}$ per cent., on an invoice of 480 yards of broad cloths, which cost in London 13s. 4d., allowing the pound sterling to be equal to \$4.84?
Ans. $516.26\frac{2}{3}$.

PROFIT AND LOSS.

78. By the terms *Profit* and *Loss* are meant the sum *gained* or *lost* in any business transaction. Profit and Loss are *calculated by finding* a certain percentage on the cost of the

article. The rules and principles which have been given are sufficient for solving questions in Profit and Loss.

EXAMPLES.

1. A man purchased a piece of cloth at \$4 per yard, and sold it at \$4.60 per yard. What did he gain per cent.?

It is plain that he gained 60 cents on the cost of one yard, or \$4. Since the gain on \$4 is 60 cents, the gain on one dollar is $\frac{1}{4}$ of 60 cents, or 15 cents. Hence, the gain is 15 per cent.

2. A man, by selling cloth at \$4.60 per yard, gained 15 per cent. What did he pay for the cloth per yard?

Since the gain is 15 per cent., the gain on one dollar is 15 cents. Hence, a quantity of cloth which he would sell for \$1.15, must have cost him \$1. Therefore, the cost of the cloth per yard is equal to as many dollars as \$1.15 is contained times in \$4.60, the selling price per yard. Now, $4.60 \div 1.15 = 4$; hence, the cloth cost \$4 per yard.

3. A man, by selling cloth at \$4.80, lost 20 per cent.; what did he pay per yard for the cloth?

Since the loss is 20 per cent., the loss on one dollar is 20 cents. Hence, a quantity of cloth which he would sell for \$0.80, must have cost him \$1. Therefore, the cost of the cloth per yard, is equal to as many dollars as \$0.80 is contained times in \$4.80, the selling price per yard. Now, $4.80 \div 0.80 = 6$; hence the cloth cost 6 dollars per yard.*

* NOTE.—From the solutions of the 2d and 3d examples, we may observe,

That the cost may be found by dividing the selling price by 1 *increased by the gain per cent., or diminished by the loss per cent.*

4. I purchased a chest of tea, containing 120 pounds of tea, at 45 cents per pound. For how much must I sell this chest of tea, in order to gain $33\frac{1}{3}$ per cent. on its cost? *Ans.* $72.

5. I paid $4.85 for a yard of broad cloth, and then sold it for $5.75. How much did I make per cent.?

Ans. $18\frac{1}{2}$ per cent., nearly.

6. What is the gain per cent. in selling a village lot for $425, that cost $300? *Ans.* $41\frac{2}{3}$ per cent.

7. If I buy butter at 14 cents per pound, and sell it for 18 cents per pound, how much do I make per cent.? What would be my gain in selling 200 dollars' worth of butter at the latter price? *Ans.* 57.14\frac{2}{7}$, $28\frac{4}{7}$ per cent.

8. I purchased 50 pounds of tea, at 75 cents per pound. How must I sell it per pound, and gain 30 per cent.?

Ans. $0.975.

9. If I buy 150 pounds of butter, at 15 cents per pound, and sell $\frac{2}{3}$ of it at a profit of 25 per cent., and the remainder at a profit of $12\frac{1}{2}$ per cent., what per cent. shall I gain on the whole? *Ans.* $20.8\frac{1}{3}$ per cent.

10. If I buy books at $7.50 per dozen, how must they be sold per copy, in order that I gain $33\frac{1}{3}$ per cent.?

Ans. 0.83\frac{1}{3}$.

11. If I buy cloth at $4.50 per yard, how must I sell it per yard, in order that I may gain 35 per cent.? *Ans.* $6.075.

12. How much per cent. do I lose in selling flour at 4.62\frac{1}{2}$ per barrel, which cost $5.50 per barrel?

Ans. 15.9 per cent., nearly.

13. If I purchase stock at $4\frac{1}{2}$ below par, and sell it at $6\frac{1}{2}$ above par, what do I gain per cent.?

Ans. $11\frac{1}{2}$ per cent., nearly.

14. What per cent. on the total amount of a merchant's sales is equivalent to 12 per cent. on the cost, if he sells his goods at a profit of $38\frac{1}{8}$ per cent.? *Ans.*

15. If I buy a cask of wine, containing 72 gallons, at $1.75 per gallon, and 10 per cent. of it afterwards leaks out; how must I sell the remainder per gallon, in order that I may gain $33\frac{1}{3}$ per cent. on the cost? *Ans.* $2.592.

16. A broker sold railroad stock to the amount of $64500, which was $7\frac{1}{2}$ per cent. advanced; what was the cost? *Ans.* $60000.

17. A manufacture finds that it costs him $8\frac{3}{4}$ cents per yard to manufacture cotton-cloth; how must he sell the cloth per yard, and make 15 per cent.? *Ans.* $0.888, nearly.

18. A publisher finds that it costs him 31 cents to manufacture a certain book; how must he sell these books per dozen, in order that he may make $33\frac{1}{3}$ per cent.? *Ans.* $4.96.

19. A grocer sold a quantity of sugar for $420.80, and made 25 per cent. on the cost. What did the sugar cost? *Ans.* $336.64.

20. A tradesman purchased a house and lot for $3000, and he made repairs on the house which cost him $456. For how much must he sell his house and lot, in order that he may make 15 per cent. on the entire cost. *Ans.* $3974.40.

ASSESSMENT OF TAXES.

79. A tax is a sum of money assessed on the person or property of a citizen for the purpose of defraying the public expenses. Taxes are generally assessed on the citizens in proportion to their taxable property. Sometimes, however, they

are assessed on the person, and then they are called *poll** taxes.

In assessing taxes, a complete inventory of all property liable to taxation must be made, and the amount of taxable property belonging to each individual paying a tax, must be ascertained. If there is any poll tax, it must be subtracted from the whole tax, and the remainder will be the tax which is to be raised on the taxable property.

Having determined the amount of tax which is to be raised on the taxable property, *divide this amount by the amount of taxable property, and the quotient will be the tax on one dollar. Then multiply the tax on one dollar, by each person's inventory, and the product will be the tax on his property. To the tax on his property, add his poll tax, if any, and the sum will be his entire tax.*

EXAMPLES.

A tax of $8000 is to be raised in a town containing 800 polls. The taxable property in the town amounts to $1480000, and the tax on each poll is $0.75. What will be Jno. Thompson's tax whose property is inventoried at $1500, and who pays for two polls.

For the solution of this question, we have the following

OPERATION.

$0.75	$8000	$ $
800	600	1480000)7400.000(0.005
$600.00	$7400	7400000

* NOTE.—The word poll is derived from a Saxon word, which signifies the *head* of a person. It is applied to all persons who are *liable to taxation.*

$\$0.005 \times 1500 = \$7.50$, and $\$0.75 \times 2 = 1.50$.

$\therefore \$7.50 + \$1.50 = \$9.00$, the tax required.

We find the amount of the poll tax to be $600, which we subtract from the whole tax, $8000, and have $7400. We then divide this remainder by $1480000, the taxable property, and find that the tax on $1 is 5 mills, or expressed decimally, $0.005. Since the tax on $1 is $0.005 on $1500, it is 1500 ×$0.005=$7.50. To this we add the tax on two polls at $0.75, and obtain $9 for the tax required.

Having found the tax on one dollar, we may facilitate the computations in finding taxes on any sum in this question by means of the following

TABLE.

The tax on	$1	is	$0.005	The tax on	$ 70	is	0.35
"	2	"	0.010	"	80	"	0.40
"	3	"	0.015	"	90	"	0.45
"	4	"	0.020	"	100	"	0.50
"	5	"	0.025	"	200	"	1.00
"	6	"	0.030	"	300	"	1.50
"	7	"	0.035	"	400	"	2.00
"	8	"	0.040	"	500	"	2.50
"	9	"	0.045	"	600	"	3.00
"	10	"	0.050	"	700	"	3.50
"	20	"	0.10	"	800	"	4.00
"	30	"	0.15	"	900	"	4.50
"	40	"	0.20	"	1000	"	5.00
"	50	"	0.25	"	2000	"	10.00
"	60	"	0.30	"	3000	"	15.00

The pupil will find no difficulty in applying this table in solving the following questions:

2. By the above table, what is the tax on $355, there being 1 poll? *Ans.* 2.525.

3. If my taxable property is $9840, and I pay for 3 polls, what is the amount of my tax? *Ans.* $51.45.

4. If my taxable property is $575, and I pay for 1 poll, what is my tax? *Ans.* $3.625.

5. If my taxable property is $2780, and I pay for 2 polls, what is my tax? *Ans.* $15.40.

6. If, when I pay for 3 polls, my tax is $17.80, what is the amount of my taxable property? *Ans.* $3110.

7. If, when I pay for 2 polls, my tax is $9.75, what is the amount of my taxable property? *Ans.* $1650.

8. If a state tax of half a mill on a dollar is levied on the property of the state, what must A pay, whose property amounts to $17840? *Ans.* $8.92.

9. In a school district a tax of $800 is to be raised for the purpose of building a school-house. If the amount of taxable property is $250000, what will be the tax on $1, and what is A's tax whose property is valued at $1800?*

Ans. { $0.0032, the tax on $1,
$5.76 A's tax.

INTEREST.

80. *Interest* is the sum which is paid by the borrower to the lender for the use of money. It is computed at a certain *per cent. per annum*; that is, the borrower pays the lender so many dollars for the use of $100 for a year. The amount of interest paid for the use of $100 for one year, is called the *rate of interest*, or the *rate per cent.* Thus, when $7 is paid for the use of $100 for one year, the rate per cent. is 7. The money lent is called the *principal*, and the interest added to the principal is called the *amount.*

* Form a table for this question.

When interest is regularly received at stated periods, or when it is computed on the principal for the whole time for which it was lent, it is called *Simple Interest.*

The rate of interest is generally established by law. The rate per cent. is not the same in all States and countries. In the New England States, the legal rate is 6 per cent., in New York it is 7 per cent., and in the other Middle States it is 6 per cent.

SIMPLE INTEREST.

81. In calculating interest, the month is generally assumed to contain 30 days, and the year 12 months, or 360 days. By placing such a value upon the month and year, it is obvious that the interest found will not be quite accurate, but it will be sufficiently accurate unless the principal is quite large.

It is plain that if we multiply the rate per cent., or the interest on one dollar for one year, by the principal, the product will be the interest on the principal for one year; and this interest multiplied by any number of years, will be the interest on the principal for that number of years.

To find the interest for months or days, we can take such fractional part of one year's interest, as is denoted by the given number of months or days.

We have, then, for finding the interest on any principal, for a given time, and at any given per cent., the following

RULE.

I. *Multiply the rate per cent., expressed as a decimal fraction, by the principal, and this product by the number of years in the given time. This last product will be the interest for the given number of years.*

II. *If there be days and months in the given time, take such fractional parts of one year's interest, as are denoted by the given number of months and days, and find the sum of these fractional parts and the interest on the principal for the given number of years. This sum will be the interest required.*

EXAMPLES.

1. What is the interest on $480 for 3 years, 5 months, and 18 days, at 7 per cent.?

OPERATION.

$480		
$0.07		
3)$33.60	Interest for 1 year.	
3		
$100.80	Interest for 3 years.	
4) 11.20	Interest for 4 months.	5 mo.
2.80	Interest for 1 month.	
1.40	Interest for 15 days.	18 days.
0.28	Interest for 3 days.	
$126.48 *Ans.*		

Since the interest on $1 for 1 year is $0.07, the interest on $480 for the same time, is 480 times $0.07=$33.60. Since the interest on the principal for 1 year is $33.60, for 4 months, or one third of a year, the interest must be $\frac{1}{3}$ of $33.60=11.20. If the interest for 4 months is 11.20, for 1 month, it is $\frac{1}{4}$ of $11.20=$2.80. Since 15 days=$\frac{1}{2}$ of a month, the interest for 15 days is $\frac{1}{2}$ of $2.80=$1.40. Again, the interest for 15 days being $1.40, the interest for 3 days, or one fifth of 15 days, is $\frac{1}{5}$ of $1.40=0.28.

2. What is the interest of $280 for 2 years, 6 months, and 24 days, at 7 per cent.? *Ans.* $50.31, nearly.

3. What is the interest of $360 for 3 years, 8 months, and 21 days, at 7 per cent.? *Ans.* $93.87.

4. What is the interest of $400 for 4 years, 3 months, and 20 days, at 6 per cent.? *Ans.* 103.33$\frac{1}{3}$.

5. What is the interest of $600 for 3 years, 2 months, and 17 days, at 8 per cent.? *Ans.* 154.266\frac{2}{3}$.

6. What is the interest of $800 for 6 years, 9 months, and 28 days, at 7 per cent.? *Ans.* 382.35\frac{5}{9}$.

7. What will $375 amount to for 4 years, 10 months, and 19 days, at 7 per cent.? *Ans.* $503.26.

8. What will $250 amount to in 6 years, 11 months, and 23 days, at 6 per cent.? *Ans.* 354.708\frac{1}{3}$.

9. What is the interest of $350.40 for 2 years and 8 months, at 4$\frac{1}{2}$ per cent.? *Ans.* $42.048.

10. What is the interest of $1600 for 3 years and 6 months, at 6 per cent.? *Ans.* $336.

11. What is the interest of $2000 for 7 years and 8 months, at 7$\frac{1}{2}$ per cent.? *Ans.* $1150.

12. What is the interest of $3000 for 10 years and 10 months, at 6$\frac{3}{4}$ per cent.? *Ans.* $2193.75.

13. What is the interest of £240 18s. 8d. for 4 years and 7 months, at 6 per cent.? * *Ans.* £66 5s. $\frac{1}{2}$d.

14. What is the interest of £60 14s. 9d. for 6 years, 8 months, and 24 days, at 7 per cent.?
Ans. £28 12s. 6d., nearly.

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* NOTE.—When the principal is in English money, reduce the shillings, pence, and farthings, to the decimal of a pound, and then proceed as in Federal money.

15. What is the interest of £340 15s. for 7 years, 8 months, and 15 days, at 6 per cent.? *Ans.* £105 6s. 6d.

16. What is the interest of \$245 for 1 year and 8 months, at 8 per cent.? *Ans.* \$32.66$\frac{2}{3}$.

17. What is the interest of \$1500 for 5 years, 5 months, and 5 days, at 5 per cent.? *Ans.* \$407.29$\frac{1}{6}$.

18. What is the interest of \$380 for 6 years, 6 months, and 19 days, at 4$\frac{1}{2}$ per cent.? *Ans.* \$112.05$\frac{1}{4}$.

19. What is the interest of \$950 for 8 years, 7 months, and 14 days, at 7 per cent.? *Ans.*

20. What is the interest of \$245 for 9 years, 10 months, and 12 days, at 8 per cent.? *Ans.* \$193.38$\frac{2}{3}$.

21. What is the interest on \$240 from Jan. 12, 1851, to March 24, 1852, the rate per cent. being 6.? * *Ans.* \$15.28.

23. What is the interest on \$500 from July 16, 1854, to Sept. 23, 1855, the rate per cent. being 7? *Ans.* \$41.513$\frac{5}{6}$.

24. What is the interest on \$650 from May 25, 1853, to October 28, 1855, the rate per cent. being 7? *Ans.* \$110.33 +.

25. A person purchases a house and lot for \$4800, and he is to pay for the property in three equal annual instalments, with interest at 7 per cent.? How much interest will he pay? *Ans.* \$672.

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* NOTE.—Find the time by the rule for the subtraction of Denominate Numbers. The following is the operation for finding the time:

Yrs.	*mo.*	*da.*
1852	3	24
1851	1	12
1	2	12

Since March is the third month, and January the first month of the year, we designate them by these numbers, respectively.

26. A man owns a house which cost him $2400, and the annual repairs on the house cost him $15. How much rent should he charge in order that he may make the interest on his money, the legal rate per cent. being 7? *Ans.* $183.

27. A person's store debt amounts to $175, and it has been on interest for 3 years, 2 months, and 18 days. What is the amount of the debt and the interest on it, the legal rate per cent. being 7? *Ans.* $214.40.

82. Since the year consists of 365 days, if we divide the rate per cent., or the interest on one dollar for one year, by 365, we shall obtain the interest on one dollar for one day. Having found the interest on one dollar for one day, we can readily form the following table, by the aid of which, we can easily compute the interest on any principal for any required time.

TABLE.

Days.	Interest.	Days.	Interest.	Days.	Interest.
1	0.00019	8	0.00153	60	0.01151
2	0.00038	9	0.00173	70	0.01342
3	0.00058	10	0.00192	80	0.01534
4	0.00077	20	0.00384	90	0.01726
5	0.00096	30	0.00575	100	0.01918
6	0.00115	40	0.00767	200	0.03836
7	0.00134	50	0.00959	300	0.05753

EXAMPLES.

1. What is the interest on $380 from May 9 to October 17, at 7 per cent.?

From the time table on page 127, we find that the number of days from May 9 to October 9, is 153. The number of

days from October 9 to Oct. 17, is 17—9=8 days. Hence, the number of days from May 9 to Oct. 17, is 153+8=161 days.

From the interest table we find that

	$0.00019	is the interest on	$1 for	1 day,
	0.01151	"	"	60 days,
	0.01918	"	"	100 days.
Hence,	$0.03088	"	"	161 days.

The interest on one dollar being $0.03088 for the given time, the interest on $386 is 380 times $0.03088, or $11.92, nearly. If we compute the interest by the rule given in the last article, we shall find it to be $11.69, nearly; hence, the error is about 22 cents.

What is the interest on $400 from December 5, 1852, to July 15, 1853, at 6 per cent.?

From December 5, 1852, to January 1, 1853, is 27 days. From the time table we find that the time from January 1 to July 1, is 181 days. From July 1 to July 15, is 15—1=14 days. Hence, the time required is 181+27+14=222 days.

From the interest table we find that

	$0.00038	is the interest on	$1 for	2 days,
	0.00384	"	"	20 days,
	0.03836	"	"	200 days,
Hence,	$0.04258	"	"	222 days.

The interest on $1, at 7 per cent., being $0.04258 for the given time, the interest on $400 is 400 times $0.04258, or $17.03, nearly. If we diminish the interest at 7 per cent. by

the *seventh* part of itself, we shall obtain the interest at 6 per cent. Hence,

$$\$17.03-\tfrac{1}{7} \text{ of } \$17.03=\$17.03-\$2.84=\$14.19$$

is the interest required.

3. What is the interest on \$600 for 250 days, at 7 per cent.? *Ans.* \$28.77.

4. What is the interest on \$840 for 210 days, at 7 per cent.? *Ans.* \$33.835

5. What is the interest on \$500 dollars for 320 days, at 8 per cent.? *Ans.* 35.80.

6. What is the interest on \$1000 for 263 days, at 7 per cent.? *Ans.* \$50.45.

7. What is the interest on \$250 from March 17 to November 17, at $4\frac{1}{2}$ per cent.? *Ans.* \$7.55.

8. What is the interest on \$450 from April 4 to June 19, at 7 per cent.? *Ans.*

9. What is the interest on \$800 from May 15 to December 19, at 7 per cent.? *Ans.* \$33.45.

10. What is the interest on \$75 from January 1 to September 1, at 7 per cent.? *Ans.* \$3.49.

11. What is the interest on \$100 for 90 days, at 7 per cent.? *Ans.* \$1.73.

12. What is the interest on \$150 for 60 days, at 7 per cent.? *Ans.* \$1.73.

13. What is the interest on \$275 for 180 days, at 7 per cent.? *Ans.* \$9.49.

14. What is the interest on \$25.83 for 275 days, at 7 per cent.? *Ans.* \$1.36.

15. What is the interest on $2000 from July 1, 1851, to October 17, 1852, at 7 per cent.? *Ans.* $181.80.*

16. What is the interest on $1500, from May 15 to December 25, at 7 per cent.? *Ans.* $64.45.

17. What is the interest on $37.50, from February 15 to September 26, at 10 per cent.? *Ans.* $2.29.

18. What is the interest on $18.45 from July 7 to November 28, at 9 per cent.? *Ans.* 0.65.

19. What is the interest on $225.67, from July 17, 1853, to May 25, 1854, at 7 per cent.? *Ans.*

20. What is the interest on $300 from September 6, 1853, to Oct. 9, 1854, at 7 per cent.? *Ans.* $22.90

A METHOD OF COMPUTING INTEREST AT SIX PER CENT.

83. This method regards the year as being composed of 12 equal parts, or months, and that each month contains 30 days.

Since the interest on one dollar, at 6 per cent., is 6 cents for one year, it follows that the interest on one dollar for 2 months is $\frac{2}{12}$ of 6 cents, or what is the same thing, $\frac{1}{6}$ of 6 cents, or 1 cent. That is, the interest on one dollar for any given number of months, at 6 per cent., is equal to as many cents as 2 is contained times in the number of months.

Again, since the interest on one dollar for 2 months, or 60 days, is 1 cent, for 6 days it is $\frac{6}{60}$, or $\frac{1}{10}$ of 1 cent, or 1 mill. That is, the interest on one dollar for any given number of days, is one-sixth as many mills as there are days.

* Note.—The pupil must recollect that the year 1852 was a LEAP YEAR. By the rule in the last article we should find the interest to be *$181.22.*

Hence, for computing the interest on any principal for a given number of months and days, we have the following.

RULE.

Divide the number of months by 2, and the number of days by 6, and call the former quotient cents, and the latter mills; and then take the sum of these two quotients for the interest on one dollar for the given time. Having found the interest on one dollar, multiply it by the principal, and the product will be the interest required.

EXAMPLES.

1. What is the interest on $76, from July 15, 1853, to Oct. 9, 1854, at 6 per cent.?

OPERATION.

To find the time.

yr.	*mo.*	*da.*
1854	10	9
1853	7	15
1	2	24

Int. on. $1 for 1 year	=$ 0.06
" " " " 2 months	= 0.01
" " " " 24 days	= 0.004
" " " the given time	=$0.074

$$\begin{array}{r} \$0.074 \\ 76 \\ \hline 444 \\ 518 \\ \hline \$5.624 \end{array}$$ *Ans.*

In this example, we first found the time, which is 1 year, 2 months, and 24 days. The interest on $1 for this time is $0.074, and by multiplying this by the principal, we obtain for the required interest, $15.624.

2. What is the interest on $245 for 2 years, 7 months, and 18 days, at 6 per cent.? *Ans.* $38.71.

3. What is the interest of $325 for 3 years, 8 months, and 21 days, at 6 per cent.? *Ans.* $72.64.

4. What is the interest of $425 for 6 years, 4 months, 15 days, at 6 per cent.? *Ans.* $162.56

5. What is the interest of $27.85 for 1 year, 2 months, and 27 days, at 6 per cent.? *Ans.* $2.075.

6. What is the interest of $164 for 5 years, 16 days, at 6 per cent.? *Ans.* $49.64.

7. What is the interest of $275 for 8 years, 3 months, and 27 days, at 7 per cent.?* *Ans.* $160.25.

8. What is the interest of $360 for 2 years, 6 months, and 12 days, at 8 per cent.? *Ans.* $72.96.

9. What is the interest of $27.63 for 1 year, 9 months, and 18 days, at 4½ per cent.? *Ans.* $2.24.

10. What is the interest of $450 for 7 years, 8 months, and 15 days, at 5 per cent.? *Ans.* $173.44.

11. What is the interest of $1600 for 5 years, 7 months, and 24 days, at 8 per cent.? *Ans.* $723.20.

12. What is the interest of $2400 for 7 years, 9 months, and 18 days, at 10 per cent.? *Ans.* $1872.

13. What is the interest of $385.47 for 6 years, 11 months, and 11 days, at 2 per cent.? *Ans.*

---

* NOTE.—Having found the interest on the principal for the given time, increase it by one-sixth of itself, and the sum will be the interest at 7 per cent.

14. What is the amount* of $27.43 for 3 years, 5 months, and 13 days, at $5\frac{1}{2}$ per cent.? *Ans.*

15. What is the amount of $385.62 for 6 months and 20 days, at 6 per cent.? *Ans.* $398.41.

16. What is the amount of $256.84 for 7 years and 8 months, at 7 per cent.? *Ans.* $394.677.

17. What is the amount of $3840 for 3 years and 10 months, at 10 per cent.? *Ans.* $5312.

18. What is the amount of $5600 for 1 year and 6 months, at 7 per cent.? *Ans.* $6188.

19. What is the amount of $75 for 2 years, 7 months, and 27 days, at 8 per cent.? *Ans.* $90.95.

20. What is the amount of $4500 for 9 months and 24 days, at $8\frac{1}{2}$ per cent.? *Ans.* $$4812\frac{3}{8}$.

PROBLEMS IN INTEREST.

84. The principal, time, rate per cent., and interest, bear such a relation to each other, that any three of them being given, the other one may be found. Hence, we have five distinct problems.

PROBLEM I.

The principal, rate per cent., and time being given, to find the interest.

The solution of this problem has been given in preceding ar-

* NOTE.—The amount may always be found by multiplying the principal by the amount of one dollar for the given time, or by adding the interest on the principal for the given time to the principal.

ticles, but the rule for its solution may be stated more briefly, as follows:

Multiply the principal by the interest on one dollar for the given time, and at the given rate per cent., and the product will be the interest required.

PROBLEM II.

The interest, time, and rate per cent. being given, to find the principal.

According to Prob. I, the interest is the *product* of the principal and the interest on one dollar for the given time, and at the given rate per cent. If we divide this product by one of its two factors, namely, the interest on one dollar for the given time, and at the given rate per cent., we shall obtain the other factor, which is the principal. Hence, for the solution of this problem, we have the following

RULE.

Divide the given interest by the interest on one dollar for the given time, and at the given rate per cent., and the quotient will be the required principal.

PROBLEM III.

The interest, time, and principal being given, to find the rate per cent.

It is obvious that as many times as the interest on the principal, at 1 per cent., for the given time, is contained in the given interest, so many times greater is the required rate per cent. than 1 per cent. Hence, for the solution of this problem, we have the following

RULE.

Divide the given interest by the interest on the principal, at 1 per cent. for the given time, and the quotient will be the required rate per cent.

PROBLEM IV.

The time, rate per cent., and amount being given, to find the principal.

The amount may always be regarded as the product of two factors, the principal and the amount of one dollar for the given time. If, then, we divide this product by one of the factors, the quotient will be the other factor. Hence, for the solution of this problem, we have the following

RULE.

Divide the given amount by the amount of one dollar, for the given time, and at the given rate per cent. The quotient will be the required principal.

PROBLEM V.

The principal, rate per cent. and interest being given, to find the time.

It is plain that the interest may be regarded as the product of the interest on the principal for one year, at the given rate, and the number of years. If, then, we divide this product by one of the factors, the quotient will be the other factor. Hence, for the solution of this problem, we have the following

RULE.

Divide the given interest by the interest on the principal for

one year, at the given rate per cent., and the quotient will be the time required in years.

When there is a decimal fraction in the quotient, it may be reduced to months and days.

EXAMPLES.

1. In what time will \$43.20 give \$4.104 interest, at 6 per cent.? *Ans.* 1 year, 7 months.

2. In what time will \$45 give \$2.10 interest, at 7 per cent.? *Ans.* 8 months.

3. In what time will \$75 give \$4.375 interest, at 7 per cent.? *Ans.* 10 months.

4. In what time will \$150 give \$10 interest, at 5 per cent.? *Ans.* 1 year, 4 months.

5. The interest on \$450 for 4 years and 18 days is \$109.35. What is the rate per cent.? *Ans.* 6 per cent.

6. The interest on \$500 for 6 years, 2 months, and 21 days is \$140.0625. What is the rate per cent.? *Ans.* $4\frac{1}{2}$ per cent.

7. What principal will amount to \$370.50 in 2 years, at 7 per cent.? *Ans.* \$325.

8. The interest on \$600 for 2 years, 8 months, and 12 days is \$97.20. What is the rate per cent.? *Ans.* 6 per cent.

9. The amount of \$850 for 10 months and 15 days is \$909.50. What is the rate per cent.? *Ans.* 8 per cent.

10. What principal will amount to \$18.9739$\frac{1}{6}$ in 6 months and 24 days, at 7 per cent.? *Ans.* \$18.25.

11. What principal will amount to \$315.75 in 9 months, at 7 per cent.? *Ans.* \$300

12. What principal will amount to \$451.50 in 1 year and 3 months, at 6 per cent.? *Ans.* \$420.

13. What principal will in 7 months and 15 days, at 6 per cent., give \$0.825 interest? *Ans.* \$22.

14. What principal will in 8 months and 9 days, at 7 per cent., give \$50? *Ans.* \$1033.06, nearly.

15. What principal will in 1 year, 4 months, and 18 days, at 8 per cent., give \$36 interest? *Ans.* \$325.30.

16. In what time will any principal double itself, at $8\frac{3}{4}$ per cent.? *Ans.* In $11\frac{3}{7}$ years.

DISCOUNT.

85. Discount is an allowance which is made for the payment of money before it becomes due.

When a debt, due at any future time, without interest, is paid before it becomes due, it is obvious that a sum which will *amount* to this debt, at the given rate per cent., and in the time which elapses from the payment of the debt till it is due, ought to cancel the debt. Such a sum is called the ***Present Worth***; and the present worth subtracted from the debt leaves the *Discount*.

For example, the present worth of a debt of \$107, due in one year from this time at 7 per cent., is \$100, since the amount of \$100 for 1 year, at 7 per cent., is \$107.

It follows from the definition of present worth, that it may be found by means of the rule under Problem IV.

EXAMPLES.

1. What is the present worth of \$300, due in 3 years, at 7 per cent.? *Ans.*

2. What is the present worth of \$450, due in 2 years and 6 months, at 6 per cent.? *Ans.*

3. What is the present worth of \$380, due in 4 months, at 5 per cent.? *Ans.*

4. What is the present worth of \$580, due in 9 months, at 7 per cent.? *Ans.* \$551.07.

5. A merchant sold goods worth \$3600, one-fourth of the amount being due in 3 months, and the remainder in 6 months. The buyer proposes to pay for the goods at the present time. What amount should he pay, interest being 7 per cent.? *Ans.* \$3493.216.

6. A merchant purchased goods to the amount of \$2540.83, on a credit of 6 months. At the expiration of 3 months he finds that he can pay for his goods. What should he pay, interest being 7 per cent.? *Ans.* \$2497.13.

7. What is the discount on \$275, due in 8 months and 24 days, the rate per cent. being 7? *Ans.* \$13.428.

8. What is the discount on \$340, due in 3 years, 6 months, and 12 days, at 6 per cent.? *Ans.* \$59.472.

9. What is the discount on \$575, due in 10 months and 18 days, at 6 per cent.? *Ans.* \$28.94

10. What is the present worth of a note of \$350 payable in 6 months, with interest at 6 per cent., when money is worth 7 per cent. interest? *Ans.*

11. What is the present worth of \$362.88, due in 8 years and 6 months, at 8 per cent.? *Ans.* \$216.

12. What is the disconnt on \$337.35, due in 7 months and 18 days, at 6 per cent.? *Ans.* \$12.35.

BANK DISCOUNT.

186. The amount charged by a bank* for the payment of money on a note before it becomes due, is called *Bank discount.* Bank discount is found by computing the interest on the amount of the note for three days more than the time stated in the note. These three days are called *days of grace.*

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* NOTE.—A bank is an incorporated institution which is established for the purpose of loaning money, receiving deposits, discounting notes, and issuing notes, called bank notes, which are redeemable in specie, at the bank.

A note is a written promise to pay a certain amount, in a specified time, to the person who owns it. The following are the usual forms of notes:

[1.]

$50 *Albany,* July 15, 1853.

For value received I promise to pay, six months after date, Wm. Perry, or order, Fifty Dollars, with interest from date.

JNO. THOMAS.

[2.]

$75 *Buffalo,* Aug. 17, 1853.

For value received I promised to pay, two months after date, Henry Harris, or bearer, Seventy-Five dollars, with interest from date.

CHARLES P. PERRINGTON.

The person who signs a note, is called the *drawer,* or *maker,* and the person who owns it, is called the *holder.*

If Wm. Perry writes his name on the back of the first note, he is said to *endorse* it, and then any person who holds the note, can, when it is due, present it to Jno. Thomas, the maker, for payment, and if he refuses to pay it, the holder may then demand payment of the endorser, Wm. Perry.

When the note is made payable to P. W., *or bearer,* the maker of the note is alone responsible for its payment.

When money is borrowed on a note from a bank, the bank deducts the discount from the amount for which the note is given, and pays over the remainder to the holder. This remainder is called the *avails*, or *proceeds* of the note.

EXAMPLES.

1. What is the bank discount on a note of $500 for 3 months and 12 days, at 6 per cent.? *Ans.* $8.75.

2. What is the bank discount on a note of $250, payable in 3 months, at 7 per cent.? *Ans.* $4.52.

3. What is the bank discount on a draft of $3750, payable in 90 days, at 6 per cent.? *Ans.* $58.125.

4. What is the bank discount on a draft of $3000, payable in 60 days, at 5 per cent.? *Ans.* $26.25

5. What is the bank discount on a draft of $2500, payable in 30 days, at 4 per cent.? *Ans.* $9.16.

6. What are the avails of a note discounted at a bank for 60 days, at 6 per cent.? *Ans.*

7. What is the bank discount on $4240, payable in 90 days, at 5 per cent.? *Ans.* $54.76.

8. What are the proceeds of a note of $575, discounted at a bank for 120 days, at 6 per cent.? *Ans* $563.21.

9. What are the proceeds of a note of $1500, discounted at a bank for 6 months, at 7 per cent.? *Ans.* $1446.63½.

10. A note of $750, payable in 8 months, is discounted at a bank, at 7 per cent. What sum is received on it? *Ans.* $714.56.

11. A note of $1200, payable in 2 years, is discounted at a bank at 6 per cent. What sum is received on it?

Ans. $1055.40.

12. What is the difference between the bank discount, and the true discount, of $107 for 1 year, at 7 per cent.?

Ans. $0.542.

87. It is often required to have a note of such an amount, that when it is discounted at the bank, the proceeds will be equal to a given sum. Hence, we have the following

PROBLEM.

It is required to find a sum, the proceeds of which, when discounted at a bank, for a specified time, at given per cent., shall be equal to a given sum.

Since the proceeds of any sum, may be found by multiplying the proceeds of one dollar by that sum, it follows that if we divide the given sum by the proceeds of one dollar, the quotient will be the sum required. Hence, we have the following

RULE.

Find the interest on one dollar for three days more than the given time, and subtract this interest from one dollar, and divide the given sum by this remainder. The quotient will be the sum required.

EXAMPLES.

1. What must be the amount of a note, so that when discounted at the bank for 60 days, at 6 per cent., the proceeds shall be $600? *Ans.* $612.55.

2. What must be the amount of a note, so that when discounted at the bank for 90 days, at 7 per cent., the proceeds shall be $1000? *Ans.* $1018.42.

3. A merchant wishes to borrow on his note for 90 days, at a bank, $2500. For what amount must the note be drawn, the rate per cent. being 6? *Ans.* $2539.40.

4. What must be the amount of a note, so that when discounted at a bank for 3 months and 21 days, at 6 per cent., the proceeds shall be $2000? *Ans.* $2038.73.

5. A man received $2400 as the avails of a note discounted at a bank for 60 days. What was the amount of the note?

Ans.

6. A man purchased a village lot for $750 cash, and sold it the same day for $950, and took a note for the amount, payable in 2 months. He had the note discounted at the bank, at 6 per cent. How much did he make by the bargain?

Ans. $190.025.

PARTIAL PAYMENTS.

88. The rule adopted by the Supreme Court of the United States for finding the amounts due on notes on which partial payments have been made, is as follows:

"*Apply the payment, in the first place, to the discharge of the interest then due. If the payment exceed the interest, the surplus goes towards discharging the principal, and the subsequent interest is to be computed on the balance of principal remaining due. If the payment be less than the interest, the surplus of interest must not be taken to augment the principal; but interest continues on the former principal until the period when the payments taken together, exceed the interest due, and*

then the surplus is to be applied towards discharging the principal; and interest is computed on the balance, as aforesaid."

This rule has been adopted by *Massachusetts*, *New York*, and nearly all the other States in the Union.

EXAMPLES.

(1.)

$500. *Homer*, July 13, 1850.

On demand I promise to pay James Parker, or order, Five Hundred Dollars, with interest at 6 per cent., for value received.

THOMAS SUMMERS.

The following payments were endorsed on this note:

May 25, 1851,	received	$20.00
July 19, 1852,	"	100.00
Aug. 18, 1853,	"	250.00
Dec. 24, 1853,	"	156.45

How much is due on this note April 6, 1854?

	Yr.	mo.	da.	Yr.	mo.	da.	Int. on $1.	Paym'ts.
Time of settlem't.	1854	4	6					
4th paym't made,	1853	12	24	0	3	12	$0.017	$156.45
3d " "	1853	8	18	0	4	6	0.021	250.00
2d " "	1852	7	19	1	0	29	0.064$\frac{5}{6}$	100.00
1st " "	1851	5	25	1	1	24	0.069	20.00
Date of note,	1850	7	13	0	10	12	0.052	

The different periods of time are readily found by subtracting each date from the one above it, as in Compound Subtraction.

Having found the different periods of time, we next find the interest on one dollar for each of these periods of time, and arrange each interest opposite to the time for which it was found. We then proceed with the work as follows:

The first principal, or the amount of the note is	$500.00
Interest on the same to July 19, 1852 * is	60.50
	560.50
The sum of the first two payments is	120.00
The amount due July 19, 1852, or the 2d principal, is	440.50
The interest from July 19, 1852, to Aug. 18, 1853, is	28.56
	469.06
The 3d payment is	250.00
The amount due August 18, 1853, is	219.06
Int. on this from August 18, 1853, to Dec. 24, 1853,	4.60
	223.66
The 4th payment is	156.45
The amount due Dec. 24, 1853, is	67.21
The int. on this from Dec. 24, 1853, to April 6, 1854, is	1.14
The amount due April 6, 1854, is	68.35

(2.)

$320. *Auburn*, July 25, 1849.

For value received, I promise to pay Jno. Thompson, or bearer, Three Hundred and Twenty dollars, on demand, with interest, at 7 per cent. WM. JONES, JR.

The following payments were endorsed on this note:

Oct. 13, 1849, $50.00.
Jan. 15, 1850, $53.45.
Dec. 18, 1851, $125.00.

What was due on this note April 6, 1852? *Ans.* 134.58

* NOTE.—We can discover by *inspection*, that the interest on the principal from the date of the note to the *first* payment is *greater* than that payment. Therefore we compute the interest from the date of the note to the time of the second payment.

(3.)

$783.67. *Syracuse*, Dec. 24, 1850.

For value received, I promise to pay Charles P. Palmer, or order, Seven Hundred and Eighty-Three Dollars and Sixty-Seven Cents, on demand, with interest at 7 per cent.

BENJAMIN HAYWOOD.

On this note the following endorsements were made:

April 1, 1851, $500.00.
July 30, 1851, $150.42.
Dec. 18, 1851, $50.73.

What was due on this note April 1, 1852? *Ans.*

(4.)

$2000 *Buffalo*, Jan. 1, 1851.

For value received, I promise to pay Jno. Spencer, or order, Two Thousand Dollars, on demand, with interest at 7 per cent.

SAMUEL HENRY.

On this note the following endorsements were made:

Nov. 28, 1851, $125.
August 1, 1852, $1875.

What was due on this note Jan. 1, 1853? *Ans.* $280.

COMPOUND INTEREST.

89. When the interest which becomes due at each stated period of time, is added to the principal, and interest is computed on this amount, as a new principal, and added to it, and so on for each period of time, the amount thus obtained is

called the amount at *Compound Interest;* and if from this amount we subtract the original principal, we shall obtain the *Compound Interest* on the principal for the given time.

The operations for computing Compound Interest by the principles of arithmetic are generally tedious, and we shall therefore give only a few examples under this head. The pupil can verify his results by means of the table which follows the examples.*

EXAMPLES.

1. What is the Compound Interest of $250 for 2 years at 7 per cent.?

OPERATION.

		$250	Principal for 1st year.
$250×0.07	=	17.50	Interest for the 1st year.
		267.50	Principal for 1st year.
$267.50×0.07	=	18.725	Principal for the 2d year.
		285.225	Amount at Com. Int. for 2 yrs.
First Principal		250.00	
		$35.225	Com. Int. for 2 years.

2. What is the compound interest of $375 for 3 years, at 5 per cent.? *Ans.* $59.109.

3. What is the compound interest of $4000 for 4 years, at 6 per cent.? *Ans.* $1049.908.

* NOTE.—Compound Interest is most readily computed by means of a table, showing the compound interest of one dollar for different periods of time, and at different rates per cent. Such a table is calculated by means of a Logarithmic Table. For the Theory and Application of Logarithms, see my "Elements of Algebra."

4. What is the compound interest of $1000 for 2 years, at 6 per cent., payable semi-annually? *Ans.* $125.509.

5. What is the amount of $300 for 2 years, at 6 per cent., payable semi-annually? *Ans.* $337.652.

6. What is the amount of $200 for 3 years, at 6 per cent., payable semi-annually? *Ans.* $238.810.

7. Find the difference between the simple interest and compound interest of $250 for 3 years, at 7 per cent.? *Ans.* $3.76.

8. At simple interest, what must be the rate per cent. on $200 for 2 years, in order that the interest may be equal to the compound interest on the same sum and for the same time, at 6 per cent.? *Ans.* 6.18 per cent.

9. What is the amount of $250 for 1 year and 6 months, at 8 per cent., the interest being payable semi-annually? *Ans.* $281.216.

10. The amount of a certain principal for 2 years, at 7 per cent., is $228.98; what is the principal? *Ans.* $200.

11. What is the present worth of $500, due in 3 years, at 6 per cent., if we take compound interest into consideration?* *Ans.* $419.81.

12. What is the compound discount on $600, due in 2 years, at 7 per cent.? *Ans.*

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* NOTE.—If money draws compound interest, the present worth of a sum payable at some future time is such a sum as will, when it is put out at compound interest for the given time and rate, amount to the debt. If from the given sum we subtract its present worth we shall obtain the *Compound* Discount.

Since the amount at compound interest may be found by multiplying the principal by the amount of one dollar at compound interest for the given time and rate, it follows that the present worth may be found by dividing the given amount by the amount of one dollar.

TABLE,

SHOWING THE AMOUNT OF $1 FOR ANY NUMBER OF YEARS NOT EXCEEDING 35, AT 3, 4, 5, 6, AND 7 PER CENT., AT COMPOUND INTEREST, THE INTEREST BEING COMPOUNDED YEARLY.

Years.	3 per cent.	4 per cent.	5 per cent.	6 per cent.	7 per cent.
1.	1.030,000	1.040,000	1.050,000	1.060,000	1.07,000
2.	1.060,900	1.081,600	1.102,500	1.123,600	1.14,490
3.	1.092,727	1.124.864	1.157,625	1.191,016	1.22,504
4.	1.125,509	1.169,859	1.215,506	1.262,477	1.31,079
5.	1.159,274	1.216,653	1.276,282	1.338,226	1.40,255
6.	1.194,052	1.265,319	1.340,096	1·418,519	1.50,073
7.	1.229,874	1.315,932	1.407,100	1.503,630	1.60.578
8.	1.266,770	1.368,569	1.477,455	1.593.848	1.71,818
9.	1.304,773	1.423,312	1.551,328	1.689,479	1.83,845
10.	1.343,916	1.480,244	1.628,895	1.790,848	1.96,715
11.	1.384,234	1.539,454	1.710,339	1.898,299	2.10,485
12.	1.425.761	1·601,032	1.795,856	2.012,196	2.25,219
13.	1.468,534	1.665,074	1.885,649	2.132.928	2.40.984
14.	1.512,590	1.731,676	1.979.932	2.260,904	2.57,853
15.	1.557,967	1.800,944	2.078,928	2.396,558	2.75,903
16.	1.604,706	1.872,981	2.182,875	2.540,352	2.95,216
17.	1.652,848	1.947.900	2.292,018	2.692.773	3.15,881
18.	1.702.433	2.025,817	2.406,619	2.854,339	3.37,293
19.	1.753,506	2.106,849	2.526,950	3.025,600	3.61,652
20.	1.806,111	2.191,123	2.653,298	3.207,135	3.86.968
21.	1.860,295	2.278.768	2.785,963	3.399.564	4.14,056
22.	1.916.103	2.369.919	2.925,261	3.603,537	4.43,040
23.	1.973,587	2.464,716	3.071.524	3.819,750	4.74,052
24.	2.032.794	2.563,304	3.225,100	4.048,935	5.07,236
25.	2.093.778	2.665,836	3.386,355	4.291,871	5.42,743
26.	2.156,592	2.772,470	3.555,673	4.549.383	5.80,735
27.	2.221.289	2.883,369	3.733,456	4.822.346	6.21.386
28.	2.287,928	2.998,703	3.920,129	5.111,687	6.64,883
29.	2.356.566	3.118.651	4.116,136	5.418.388	7.11,425
30.	2.427,262	3.213,398	4.321,942	5.743,491	7.61,225
31.	2.500.080	3.373,133	4.538,039	6.088,101	8.14,571
32.	2.575.083	3.508,059	4.764,941	6.453,386	8.71.527
33.	2.652,335	3.648,381	5.003,189	6.840,590	9.32,533
34.	2.731,905	3.794,316	5.253,348	7.251,025	9.97,811
35.	2.813,862	3.946,089	5.516,015	7.686,087	10.6,765

CHAPTER X.

ANALYSIS.

90. The principles which have thus far been developed are sufficient for solving the most important and practical arithmetical questions.

The process of reasoning which we employ in obtaining, without the application of any particular rule, the solution of an arithmetical question or problem, is called *Analysis.*

EXAMPLES.

1. If 18 yards of cloth cost 54 dollars, what will 75 yards cost at the same rate?

OPERATION.

$$\$\frac{\overset{3}{\cancel{54}}}{1}\times\frac{1}{\cancel{18}}\times\frac{75}{1}=\$225\ \textit{Ans.}$$

Analysis.—Since 18 yards cost 54 dollars, 1 yard will cost *one eighteenth* of 54 dollars, and 75 yards will cost 75 times as much as 1 yard costs, or 75 times *one eighteenth* of 54 dollars. By performing the multiplication, as in the multiplication of fractions, we obtain for the product, 225. Hence, if 18 yards cost 54 dollars, 75 yards will cost 225 dollars.*

* NOTE.—In solving a question by analysis, the pupil must determine from the terms which are given, the value of *one* of the terms

2. If $\frac{5}{8}$ of a yard of cloth cost \$3.75, what will $\frac{7}{16}$ of a yard cost?

OPERATION.

$$\$\ \frac{\overset{0.75}{\cancel{3.75}}}{1}\times\frac{8}{\cancel{5}}\times\frac{7}{16}=\$42\ \textit{Ans.}$$

Analysis.—Since 5 *eighths* of a yard cost \$3.75, 1 eighth will cost 1 *fifth* of \$3.75, and 8 *eighths*, or a yard, will cost 8 times as much as 1 yard costs, or *eight fifths* of \$3.75. Now, if a yard costs *eight fifths* of \$3.75, $\frac{7}{16}$ of a yard will cost $\frac{7}{16}$ of $\frac{8}{5}$ of \$3.75.

3. If 18 men can do a certain work in 12 days, in how many days can 8 men do the same work?

OPERATION.

$$\text{Days. } \frac{\overset{3}{\cancel{12}}}{1}\times\frac{\overset{9}{\cancel{18}}}{1}\times\frac{1}{\underset{2}{\cancel{8}}}=27 \text{ days.}$$

Analysis.—Since 18 men can do the work in 12 days, 1 man will require 18 times 12 days to do the same work. Again, since 1 man requires 18 times 12 days to do the work, 8 men will require 1 *eighth* of 18 times 12 days. 1 *eighth* of 18 times 12 days is equal to 27 days. Hence, if 18 men can do a certain work in 12 days, 8 men can do the same work in 27 days.

4. A man spent $\frac{1}{4}$ of his money for a house, $\frac{1}{10}$ for a horse, and then had \$2800 left. How much money did he have at first.

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of demand. Thus, in the above example, we first find the cost of *one* yard, and then multiply this cost by 75, which is the term of demand.

OPERATION.

$$\frac{1}{4}+\frac{1}{10}=\frac{7}{20}.\quad \frac{20}{20}-\frac{7}{20}=\frac{13}{20}.\quad \$\frac{\overset{200}{\cancel{2600}}}{1}\times\frac{20}{\cancel{13}}=\$4000.$$

Analysis.—By adding $\frac{1}{4}$ and $\frac{1}{10}$, we find that he spent $\frac{7}{20}$ of his money for the house and horse; hence $\frac{20}{20}-\frac{7}{20}=\frac{13}{20}$, is the part of his money which he had left, Therefore $\frac{13}{20}$ of his money is equal to \$2600. Since $\frac{13}{20}$ of his money is equal to \$2600, $\frac{1}{20}$ of his money is $\frac{1}{13}$ of \$2600, and $\frac{20}{20}$ of his money is equal to 20 times $\frac{1}{13}$ of \$2600, or $\frac{20}{13}\times\$\frac{2600}{1}=\$4000$.

5. A can chop a cord of wood in 6 hours, B in 4 hours, and C in 8 hours. In how many hours can the three, working together, chop 78 cords?

OPERATION.

$$\frac{1}{6}+\frac{1}{4}+\frac{1}{8}=\frac{13}{24}.\quad \text{Hours.}\ \frac{1}{1}\times\frac{24}{\cancel{13}}\times\frac{\overset{6}{\cancel{78}}}{1}=144 \text{ hours.}$$

Analysis.—Since A can chop a cord of wood in 6 hours, B in 4 hours, C in 8 hours, in 1 hour A can chop $\frac{1}{6}$ of a cord, B $\frac{1}{4}$ of a cord, and C $\frac{1}{8}$ of a cord. Hence, the three together can chop in 1 hour $\frac{1}{6}+\frac{1}{4}+\frac{1}{8}=\frac{13}{24}$ of a cord. Now, since they can chop $\frac{13}{24}$ of a cord in 1 hour, they can chop $\frac{1}{24}$ of a cord in $\frac{1}{13}$ of an hour, and to chop a whole cord, or $\frac{24}{24}$, they will require 24 times $\frac{1}{13}$ of 1 hour, or $\frac{24}{13}$ of 1 hour. Again, since they can chop 1 cord in $\frac{24}{13}$ of 1 hour, they can chop 78 cords in 78 times $\frac{24}{13}$ of 1 hour, or 144 hours.

6. A merchant mixes 48 pounds of sugar, worth 8 cents per pound, with 72 pounds worth 10 cents per pound. How can he afford to sell a pound of the mixture?

OPERATION.

$$\begin{array}{rr} 48\times \ 8 \text{ cents} & =\$3.84 \\ \underline{72}\times 10 \text{ cents} & =\underline{\$7.20} \\ 120 & \$11.04 \end{array}$$

$$\$11.04\div 120=\$0.092 \quad \textit{Ans.}$$

Analysis.—The value of 48 pounds of sugar, at 8 cents per pound, is \$3.84, and the value of 72 pounds at 10 cents per pound, is \$7.20. Now, 48 lbs+72 lbs.=120 lbs., and \$3.84+\$7.20=\$11.04. Hence, we see that the mixture contains 120 lbs. of sugar which is worth \$11.20. Since 120 lbs. are worth \$11.20, 1 lb. is worth $\frac{1}{120}$ of \$11.04=\$0.092. Therefore he can afford to sell the mixture at \$0.092 per pound.

7. If three painters painted a house in 24 days, by working 10 hours per day, in how many days could six painters have painted the same house, by working 12 hours per day?

OPERATION.

$$\text{Days. } \frac{\overset{4}{\cancel{24}}}{1}\times\frac{\cancel{3}}{\cancel{6}}\times\frac{10}{\cancel{12}}=10 \text{ days.}$$

Analysis.—Since 3 painters can paint the house in 24 days, by working 10 hours per day, 1 painter can paint the house in 3 times 24 days, and since 1 painter can paint the house in 3 times 24 days, 6 painters can paint the house in $\frac{1}{6}$ of 3 times 24 days, or in $\frac{3}{6}$ of 24 days. Now if 6 painters require $\frac{3}{6}$ of 24 days to paint the house, when they work 10 hours per day, they will require 10 times $\frac{3}{6}$ of 24 days, when they work 1 hour per day, and when they work 12 hours per day, they will require $\frac{1}{12}$ of as many days as they do when they work 1 hour

per day, that is $\frac{1}{12}$ of 10 times $\frac{3}{6}$ of 24 days, or $\frac{10}{12}$ of $\frac{3}{6}$ of 24 days, or 10 days.*

8. Two persons form a partnership in trade, with a capital of \$4800. The first contributed \$3000 of the capital, and the second, the remainder. They gained \$1200. What is each one's share?

OPERATION.

$$\begin{array}{r} \$4800 \\ 3000 \\ \hline \$1800 \end{array} \qquad \left.\begin{aligned} &\frac{\$\cancel{1200}}{1}\times\frac{\overset{750}{\cancel{\$000}}}{\underset{4}{\cancel{4800}}}=\$750\text{, the first one's gain.} \\ &\frac{\$\cancel{1200}}{1}\times\frac{\overset{450}{\cancel{1800}}}{\underset{4}{\cancel{4800}}}=\$450\text{, the second one's gain.} \end{aligned}\right\}\textit{Ans.}$$

Analysis.—Since the gain on \$4800 is \$1200, the gain on \$1 is $\frac{1}{4800}$ of \$1200, and the gain on \$3000 is 3000 times the gain on \$1, or 3000 times $\frac{1}{4800}$ of \$1200, or $\frac{3000}{4800}$ of \$1200 = \$750. Hence, the first one's gain is \$750. By similar reasoning, we find that the second one's gain is \$450.

The solution of this question evidently requires that the gain, \$1200, shall be divided into two parts, which shall be to each other as 3000 to 1800. By dividing each of these two numbers by 600, we find that they are to each other as 5 to 3. Hence, we may obtain the solution of the question by dividing \$1200 into two parts, which are to each other as 5 to 3.

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* NOTE.—In examples like this, and in many others, the pupil should only *indicate*, by means of the proper signs, the operations of multiplication and division which arise from their solution. In this manner, he can often render the solution of a question very easy and beautiful, which would be very tedious when conducted by the ordinary methods.

Now, this thing may be done by dividing \$1200 into 8 equal parts, and then taking 5 of these parts for one of the parts required, and 3 of them for the other required part. Hence, $\$1200 \div 8 \times 5 = \$750$, the first one's gain; and $\$1200 \div 8 \times 3$ \$450, the second one's gain.

9. A merchant has two kinds of wine: the first kind is worth 12 shillings per gallon, and the second kind is worth 7 shillings per gallon. How many gallons of each kind must he use in order to form a mixture worth 9 shillings per gallon?

Analysis.—If he sells a gallon, of each kind for 9 shillings, the price per gallon of the mixture, he will *lose* 3 shillings on the gallon of the first kind, and *gain* 2 shillings on the gallon of the second kind. Since the *loss*, in selling 1 gallon of the first kind, is 3 shillings, the *loss* in selling $\frac{1}{3}$ of a gallon is 1 *shilling*. Further, since the gain, in selling 1 gallon of the second kind, is 2 shillings, the gain in selling $\frac{1}{2}$ of a gallon of the second kind is 1 *shilling*. Therefore, the *loss*, in selling $\frac{1}{3}$ of a gallon of the first kind, is equal to the *gain* in selling $\frac{1}{2}$ of a gallon of the second kind. Hence, in forming the mixture, he can use $\frac{1}{3}$ of a gallon of the first kind as often as he use $\frac{1}{2}$ of a gallon of the second kind. The gain will then *counterbalance* the loss.

Since two fractions having a common denominator, are to each other as their numerators, we will reduce $\frac{1}{3}$ and $\frac{1}{2}$ to equivalent fractions, having a common denominator. We observe that $\frac{1}{3}=\frac{2}{6}$, and that $\frac{1}{2}=\frac{3}{6}$. Hence, in forming the mixture he may use 2 gallons of the first kind as often as he uses 3 gallons of the second kind. We may observe here that, the quantities of wine which he uses of each kind in forming the mixture, are to each other *inversely* as the differences be-

tween the prices of the two kinds of wine, and the mean price.

10. A merchant has three kinds of wine. The first kind is worth 8 shillings per gallon, the second, 9 shillings per gallon, and the third, 15 shillings per gallon. How can he form a mixture from these three kinds, so that the mixture may be worth 11 shillings per gallon?

Analysis.—By a reasoning similar to that employed in the analysis of the last example, we find that he can form a mixture of the *first* and *third* kinds of wine, which may be sold at the *mean* rate, by taking 4 gallons of the first kind, and 3 gallons of the third kind.

In a similar manner, we may find that he can form a mixture of the *second* and *third* kinds which may be sold at the *mean* rate, by using 4 gallons of the second kind, and 2 gallons of the third kind.

Now, if *these two mixtures* be mixed, it is plain that a third mixture will be formed, which may be sold at the mean rate. This third mixture will contain 4 gallons of the first kind, 4 of the second kind, and 3+2, or 5 gallons, of the third kind. *

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* NOTE.—From the analyses which have been given of the 9th and 10th examples, we may derive, for the solution of all similar questions, the following

RULE.

I. *Arrange the prices of the several ingredients which are to form the mixture, in a vertical column, and write the mean price on the left of this column. Join the price of each ingredient which is less than the mean price, with one or more prices which exceed the mean price of the compound; and connect each price greater than the mean price, with one or more that are less than the mean price.*

II. *Write the difference between the price of each ingredient and the*

11. A merchant wishes to mix 32 pounds of tea at 36 cents per pound, with some at 48 cents, and some at 72 cents. How many pounds of each kind must he use to form a mixture worth 56 cents per pound?

OPERATION.

$$56\left\{\begin{array}{l|l}36 & 16\\ 48 & 16\\ 72 & 20+8=28.\end{array}\right.$$

Analysis.—By the rule given in the note, we find that the proportional quantities which he must use, are 16 pounds of the first kind, 16 pounds of the second kind, and 28 pounds of the third kind; that is, as often as he uses 1 pound of each of the first two kinds, he must use $\frac{28}{16}$ of a pound of the third kind. Hence, if he uses 32 pounds of the first kind, he must use the same number of the second kind, and $\frac{28}{16}$ of 32 pounds, or 56 pounds of the third kind of tea.

12. A grocer has three kinds of sugar. The first kind is worth 7 cents per pound; the second, 8 cents; the third, 11 cents. How many pounds must he use of each, in forming a mixture of 350 pounds, worth 10 cents per pound?*

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mean price, opposite to the price or prices, with which it is connected. If there is only one of these differences opposite to the price of any ingredient, it will express the quantity which must be used of this ingredient in forming the mixture, but if there are more than one, their sum will express the required quantity.

* NOTE.—The 9th, 10th, 11th, and 12th examples, are questions in what is called "*Alligation*" by most authors. The better term of "*Medial Proportion*" has been proposed by Mr. Dodd, in his Arithmetic, since, in questions like the preceding, we are required to find the *proportional* parts that must be used of two or more ingredients, in order that the compound may have a *mean* value. The word Alligation is derived from the Latin verb *alligo*, which signifies to *bind*, or *tie*, and it refers to the manner of connecting the numbers by lines.

OPERATION.

$$10\left\{\begin{array}{l|l} 7 & 1 \\ 8 & 1 \\ 11 & 3+2=5 \end{array}\right.$$

Analysis.—Here we find that the proportional quantities are, 1 pound of the first kind, 1 pound of the second, and 5 pounds of the third. The sum of these proportional quantities is 7. Hence, $\frac{5}{7}$ of the mixture must be composed of the third kind, $\frac{1}{7}$ of it is composed of the second kind, and $\frac{1}{7}$ of it is composed of the first kind. That is,

$\frac{5}{7}\times 350$ lbs. $=250$ lbs., the number of pounds of the 3d kind.
$\frac{1}{7}\times 350$ lbs. $=$ 50 lbs. " " " 2d "
$\frac{1}{7}\times 350$ lbs. $=$ 50 lbs. " " " 1st "

13. A merchant in New York wishes to remit to London £480. The direct exchange between New York and London is 4.84 for £1; but the direct exchange between London and Paris is 24 francs for £1; between Paris and Hamburg it is 19 francs for 10 marcs banco; and between Hamburg and London it is 14 marcs banco for £1. If the expenses of transmitting money by the circuitous route and the direct route, are the same, had the merchant in New York better transmit his money by the direct route, or by the circuitous one through Paris and Hamburg?

OPERATION.

$$\$4.86\times 4850=23571.$$

$$\$\frac{\overset{9}{\cancel{18}}}{\underset{\underset{10}{\cancel{20}}}{\cancel{100}}}\times\frac{19}{1}\times\frac{14}{\cancel{10}}\times\frac{\overset{97}{\cancel{4850}}}{1}=\$23221.80.$$

Analysis.—Since the direct exchange between New York

and London is \$4.86, the merchant in New York would have to remit to London, in order to pay a debt of £4850, 4850 times \$4.86, or \$23571. Let us now determine how many dollars he would have to remit by the circuitous route, in order to cancel his debt in London.

Since the direct exchange between New York and Paris is 1 franc for 18 cents, or $\frac{18}{100}$ of a dollar, the exchange for 19 francs is 19 times $\frac{18}{100}$ of a dollar. Since 19 francs, or 19 times $\frac{18}{100}$ of a dollar, may be exchanged for 10 marcs banco, the exchange for 1 marc banco is $\frac{1}{10}$ of 19 times $\frac{18}{100}$ of a dollar, and the exchange for 14 marcs is 14 times $\frac{1}{10}$ of 19 times $\frac{18}{100}$ of a dollar. But 14 marcs banco may be exchanged for £1; hence, the exchange for £4850 is 4850 times $\frac{14}{10}$ of 19 times $\frac{18}{100}$ of a dollar, or \$23221.80. If we subtract this sum from the amount which he remitted by the direct route, we shall find how much he gained by remitting his money by the circuitous route.*

* NOTE.—Examples of this kind are generally solved by what is denominated the "chain rule," but there appears to be no necessity for having a separate rule for solving such examples. This example would be arranged, in most arithmetics, under the head of "Arbitration of Exchange," which is a method of finding the amount of money that will pay a foreign debt, when the money is remitted through the agency of brokers in different countries. If we let n=the required number of dollars, we shall obviously have the following equations:

$$\begin{aligned} \text{franc } 1 &= \$\tfrac{18}{100} \\ \text{Marcs banco } 10 &= 19 \text{ francs} \\ \pounds 1 &= 14 \text{ marcs banco} \\ \text{dollars } n &= \pounds 4850 \end{aligned}$$

By taking the product of these equations, we have,

$$10n = 4850 \times 14 \times 19 \times \$\tfrac{18}{100};$$

or, dividing by 10, $n = 485 \times 14 + 19 \times \$\frac{18}{100} = \$23221.80$.

From this solution, the *chain rule* might be deduced. The student

14. **Two persons, A and B, hire a pasture for $24. A has in the pasture 8 horses for 6 weeks, and B 10 horses for 12 weeks. What had each ought to pay ?**

OPERATION.

$$\begin{array}{lrcrcr} A & 6 & \times & 8 & = & 48 \\ B & 12 & \times & 10 & + & 120 \\ \hline & & & & & 168 \end{array}$$

$\$30 \times \frac{48}{168} = \$8\frac{4}{7}$, A's share. $\$30 \times \frac{120}{168} = \$21\frac{3}{7}$, B's share.

Analysis.—The pasturage of 8 horses for 6 weeks is equivalent to the pasturage of 1 horse for 8 times 6 weeks, or 48 weeks. The pasturage of 10 horses for 12 weeks is equivalent to the pasturage of 1 horse for 10 times 12 weeks, or 120 weeks. Hence, A and B together had the pasturage of 1 horse for $120+48=168$ weeks. Hence, A must pay $\frac{48}{168}$ of $30, or $8\frac{4}{7}$, and B must pay $\frac{120}{168}$ of $30, or $21\frac{3}{7}$.

15. A merchant purchased a bill of goods amounting to $2400, of which $400 is due in 4 months, $1200 in 6 months, and the remainder in 8 months, without interest. What is the proper time for the payment of the whole at once, money being worth 7 per cent. ?

OPERATION.

$\$\ 400 \div 1.02\frac{1}{3} = \$\ 390.879$, prest. worth of $ 400 due in 4 mo.
$\$1200 \div 1.03\frac{1}{2} = \$1159.421$, " $1200 " 6 mo.
$\$\ 800 \div 1.04\frac{2}{3} = \$\ 764.331$, " $ 800 " 8 mo.
$2314.631.

$\$2400 - \$2314.631 = \$85.369$.

$\$2314.631 \times 0.07 = \$162.02417$, Int. on $2314.631 for 1 year.
$85.369 \div 162.024 = 0.527$, nearly.
0.527 *years* = 6 *months* and 10 *days*, nearly.

Analysis.—We find that the sum of the present worths of

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who has an acquaintance with simple equations in Algebra, may prefer this solution to the other.

the several payments is \$2314.631. The whole debt \$2400 — \$2314.631 = 85.369. Hence, \$2400 must be paid in such a time as \$2314.631 will gain 85.369 interest at 7 per cent. By Problem 5, page 193, we find this time to be 6 *months* and 10 *days*, nearly.*

16. A debt of \$800 is due in 8 months; 4 months before it is due \$300 is paid; and 2 months before it is due \$200 is paid. What is the equitable time for paying the remainder?

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* NOTE.—Examples similar to the last are generally arranged under the rule for the *Equation of Payments*. *Equation of Payments*, then, is the process of finding the *mean*, or *equitable*, time at which several sums, payable at different times, may be paid. From the *analysis* an accurate rule may be deduced for solving examples in *Equation of Payments*, but when the periods of time are short, and the payments are small, it is better to use the following rule, since it is sufficiently accurate under these circumstances.

RULE.

Multiply each payment by the time of its credit, and then divide the sum of these products by the sum of the several payments: the quotient will be the time required.

For the solution of the last example by this rule, we have the following

OPERATION.

$$\begin{array}{rcrcl} 400 & \times & 4\ mo. & = & 1600\ mo. \\ 1200 & \times & 6\ mo. & = & 7200\ mo. \\ 800 & \times & 8\ mo. & = & 6400\ mo. \\ \hline 2400 & & & & 15200\ mo. \end{array}$$

$$15200 \div 2400 = 6\tfrac{1}{4}\ mo. = 6\ mo.\ 7\tfrac{1}{2}\ da.$$

Explanation.—The credit on \$400 for 4 *mo.* is equal to the credit on \$1 for 1600 *mo.*; the credit on \$1200 for 6 *mo.* is equal to the credit on \$1 for 7200 *mo.*; and the credit on \$800 for 8 months is equal to the credit on \$1 for 6400 *mo.* Hence, he can have a credit on \$1 for 1600 *mo.* + 7200 *mo.* + 6400 *mo.* = 15200 *mo.*, and on \$2400 he can have a credit of $\frac{1}{2400}$ of 15200 *mo.*, or 6 *mo.* and $7\frac{1}{2}$ *da.*

The rule implies the supposition that the discount on any sum for

OPERATION.

$$\begin{aligned} 300 \times 4\ mo. &= 1200\ mo. \\ 200 \times 2\ mo. &= \underline{\ 800\ mo.} \\ & 2000\ mo. \end{aligned}$$

$$2000\ mo. \div 300 = 6\tfrac{2}{3}\ mo.$$

Analysis.—A credit on $300 for 4 *mo.*, and a credit on $200 for 2 *mo.*, are the same as a credit on $1 for 2000 *mo.*, and the balance due, which is $300, must have a credit of $\frac{1}{300}$ of 2000 *mo.* $= 6\frac{2}{3}$ *mo.* beyond the 8 months.

17. If 15 bushels of oats cost $4.50, what will be the cost of 48 bushels? *Ans.* $14.40.

18. If $2\frac{3}{4}$ yards of cloth cost $22, what will be the cost of 45 yards? *Ans.* $360.

19. If 17 tons of hay cost $154.70, what will 25 tons cost? *Ans.* $227.50.

20. If 9 bushels of wheat make 2 barrels of flour, how many bushels will make 45 barrels of flour? *Ans.* $202\frac{1}{2}$.

21. If $\frac{7}{8}$ of a yard of cloth cost $2.73, what will $12\frac{3}{4}$ yards cost? *Ans.* $39.78.

22. If 12 dozen of books cost $66, what will 50 dozen cost? *Ans.* $275.

23. If a man can perform a certain piece of work in 45 days,

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a given time is equal to the interest on that sum for the same time and at the same rate. This supposition being wrong, the rule itself is incorrect. When there are two equal payments due at different times, we shall find by the rule, that one will be paid a certain time *before* it is due, and the other will be paid the *same* time *after* it is due. To find the *real* value of these payments, we must obtain the *amount* of the former, and add it to the present worth of the latter. It is obvious that this sum is not equal to the sum of the two payments. But it should be if the rule is correct.

by working 10 hours per day, how many days will he require to do the same work, if he work 12 hours per day? *Ans.* $37\frac{1}{2}$ days.

24. It takes 32 yards of carpeting that is $1\frac{1}{4}$ yards wide to carpet a floor. How many yards of carpeting that is $\frac{7}{8}$ of a yard wide, will it take to carpet the same floor? *Ans.* $45\frac{5}{7}$ yards.

25. If $\frac{3}{8}$ of a ship is worth $15000, what is $\frac{5}{12}$ of it worth? *Ans.* $$16666.66\frac{2}{3}$.

26. If the interest on $100 for 12 months is $7, what is the interest on $100 for 8 months and 15 days, if we reckon the month at 30 days? *Ans.* $4.958.

27. How many hats at the rate of $36.50 per dozen, can be bought for $511? *Ans.* 168.

28. If the interest on $100 for 12 months is $7, how long must $100 be on interest to gain $45? *Ans.* $77\frac{1}{7}$ months.

29. When A has travelled 68 days at the rate of 32 miles per day, B, who had travelled 48 days, overtook him. How may miles per day did B travel, allowing that both started from the same place? *Ans.* $45\frac{1}{3}$ miles.

30. An estate of $5400 is to be divided among three persons, A, B, and C; A is to receive $5 as often as B receives $7, and C is to receive $3 as of often as A receives $1. What is each one's share? *Ans.* A $1000; B $1400; C $3000.

31. A man sold shoes at $1.50 per pair, and lost 30 per cent. How must he sell them per pair, in order to gain 25 per cent? *Ans.* $2.68 nearly.

32. A man paid $6.45 for 688 feet of lumber. What will 12800 feet cost at the same rate, and what is the price per hundred? *Ans.* $120; the price per hundred is $$0.93\frac{3}{4}$.

33. What number is that, $\frac{7}{8}$ of which exceeds $\frac{2}{5}$ of it by 6080? *Ans.* 12800.

34. An acre of coal 2 feet thick yields 3000 tons; and one 5 feet thick 8000. How many acres of 5 feet thick would give the same quantity as 48 of 2 feet thick? *Ans.* 18.

35. A can walk from Buffalo to Williamsville in 5 hours, and B can walk the same distance in 4 hours. If they start at the same time, the one at Buffalo, and the other at Williamsville, and walk towards each other, how many hours will elapse before they meet? *Ans.* $2\frac{2}{9}$ hours.

36. A man bought a certain number of knives at the rate of 2 for a dollar, and an equal number at the rate of 3 for a dollar. He then sold his knives at the rate of 8 for 5 dollars, and by this means gained 15 dollars. How many knives did he buy? *Ans.* 72.

37. Two carpenters, A and B, built a house for $2400. A labored on the house 240 days, and advanced $700 for the purchase of building materials, and for other expenses; B labored 260 days, and advanced $800 for buying building materials. How much must each have of the $2400?

Ans. A $1132; B $1268.

38. What should I pay a mason for laying 3640 bricks, at the rate of $6.25 per thousand? *Ans.* $22.75.

39. A can do a piece of work in 3 days, B in 6 days, and C in 4 days. How many days will the three require to do the work, when they work together? *Ans.* $1\frac{1}{3}$ days.

40. A cistern containing 1200 gallons, has a discharging and a receiving pipe. The discharging pipe lets out 1 gallon in 6 minutes, and the receiving pipe lets in 1 gallon in 4 minutes. In what time can the cistern be filled, if both pipes are open? *Ans.* 10 days.

41. If a piece of cloth 3 feet long, and 3 feet wide, make 1

square yard, how long must a piece of cloth be, in order that it may contain 1 square yard, when it is $2\frac{5}{8}$ feet wide?

Ans. $3\frac{3}{7}$ feet.

42. A farmer being asked how many sheep he had, replied that he had them in five fields; in the first he had $\frac{1}{4}$, in the second $\frac{1}{6}$, in the third $\frac{1}{8}$, in the fourth $\frac{1}{12}$, and in the fifth 450. How many sheep had he? *Ans.* 1200.

43. Out of a cask which had leaked away $\frac{2}{5}$, 48 gallons were afterwards drawn, and then the cask was found to be $\frac{7}{12}$ full. How many gallons did the cask contain?

Ans. 2880 gallons.

44. I once had \$42 in my possession; of this I gave away a certain sum, and there still remained three times as much as I gave away. How much did I give away? *Ans.* \$$10\frac{1}{2}$.

45. A dying man bequeaths in his will the half of his property to his wife, one-sixth to each of his two sons, the twelfth part to his servant, and the remaining \$600 to the poor. What was the amount of his property? *Ans.* \$7200.

26. There are two numbers whose sum is 96, and one of which is greater than the other by 16. What are these numbers? *Ans.* 40 and 56.

47. The rent of an estate is this year greater by 8 per cent. than it was last year. This year's rent is \$1890. What was the rent of last year?* *Ans.* \$1750.

48. Two merchants having left off business, their profits, amounting to \$1200, are to be divided between them in such a manner, that one partner receives only half as much as the other, exclusive, however, of \$50 for his labor. How much does each receive? *Ans.* The one \$$766\frac{2}{3}$, the other \$$433\frac{1}{3}$.

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* By the question, 1890 must be $\frac{108}{100}$ of the rent for the last year.

49. A fountain has 4 faucets, A, B, C, and D, and under it stands a cistern, which can be filled by A in 6, by B in 8, by C in 10, and by D in 12 hours? the cistern has 4 faucets, E, F, G, and H; and it can be emptied by E in 6, by F in 5, by G in 4, and by H in 3 hours. Suppose that the cistern is full of water, and that the 8 faucets are all open, in what time will it be emptied? *Ans.* $2\frac{2}{19}$ hours.

50. In a mixture of spirits and water, $\frac{1}{2}$ of the whole plus 25 gallons was spirits, and $\frac{1}{3}$ of the whole minus 5 gallons was water. How many gallons were there of each?

Ans. 85 of spirits, 35 of water.

51. If 8 men can mow 64 acres in 8 days, in how many days can 12 men mow 7 times this quantity. *Ans.* $37\frac{1}{3}$ days.

52. How many bushels of wheat at $87\frac{1}{2}$ cents per bushel, should be exchanged for 63 yards of cloth, at $2.50 per yard?

Ans. 180 bushels.

53. What number is that from which if we subtract $\frac{3}{8}$ of itself, and $\frac{3}{5}$ of the remainder, 62 will remain? *Ans.* 248.

54. If the interest on $450 for 2 years and 6 months is $67.50, what is the interest on $360 for 3 years and 2 months?

Ans. $68.40.

55. If the interest on $100 for 1 year is $7, what is the interest on $275 for 1 year and 8 months? *Ans.* $$32.08\frac{1}{3}$,

56. If $100 gain $6 in 12 months, what principal will gain $76.50 in 18 months? *Ans.* $850.

57. If $100 gain $6 in 12 months, in what time will $1200 gain $180? *Ans.* 2 years and 6 months.

58. If $650 gain $175.50 in 4 years and 6 months, what will $100 gain in 1 year? *Ans.* 6 dollars.

59. A carpenter hired a certain number of journeymen, and

one-half as many apprentices. He paid each journeyman \$1.50 per day, and each apprentice \$0.37½. The daily wages of all amounted to \$13.50. How many were there of each?

Ans. 8 carpenters, 4 apprentices.

60. At 7 per cent., in what time will the interest on \$800 be the same as the interest on \$600 for 2 years and 8 months?

Ans. 2 years.

61. If 12 ounces of wool make 2½ yards of cloth 6 quarters wide, how much wool will make 150 yards 4 quarters wide?

Ans. 30 pounds.

62. If 18 masons can lay 96000 bricks in 12 days, by working 10 hours per day, how many bricks can 24 masons lay in 36 days, by working 12 hours per day? *Ans.* 460800.

63. Two men, A and B, are at the opposite sides of a wood 1780 yards in circumference. They begin to go around at the same time, and in the same direction: A travelling at the rate of 76 yards in 5 minutes, and B at the rate of 30 yards in 2 minutes. How many times must each go around the wood, before A overtakes B? *Ans.* A, 38 times; B, 37½ times.

64. If the interest on \$100, for 1 year, at 7 per cent., is \$7, what is the interest on \$250 for 2¼ years, at 8 per cent.?

Ans. \$45.

65. A wall was to be built 700 yards long in 29 days; after 12 men had been employed on it for 11 days, it was found that they had built 220 yards. How many more men must be put on to finish it in the given time? *Ans.* 4 men.

66. Divide \$2100 dollars among A, B, and C, so that A may have \$100 more than B, and B \$400 more than C. How much must each have?

Ans. A, \$900; B, \$800; C, \$400.

67. If it take 12 yards of cloth that is $\frac{7}{8}$ of a yard wide to make a cloak, how many yards of lining, that is $\frac{3}{4}$ of a yard wide, will it take to line the cloak; and what will the lining cost at \$1.62$\frac{1}{2}$ per yard? *Ans.* 14 yards; cost \$22.75.

68. The hair-spring of a watch weighs about the tenth of a grain, and costs about \$2.50. How much would be the cost of a pound of crude iron, made into steel, and then into hair-springs, if, after deducting waste, there are obtained from the iron about 7000 grains of steel? *Ans.* \$175000.

69. If 18 men can build a brick wall 360 feet long, 3 feet thick, and 8 feet high, in 400 days, in how many days can 24 men build a wall 240 feet long, 1$\frac{1}{2}$ feet thick, and 6 feet high? *Ans.* 75 days.

70. If 18 compositors can set 360 pages of types, each page containing 50 lines, and each line 45 letters, in 6 days of 9 hours, how many compositors will be required to set 480 pages of 75 lines each, and 40 letters in each line, in 12 days of 12 hours? *Ans.* 12.

71. If 25 pears can be bought for 10 lemons, and 28 lemons for 18 pomegranates, and 1 pomegranate for 48 almonds, and 50 almonds for 70 chesnuts, and 108 chesnuts for 2$\frac{1}{4}$ cents, how many pears can I buy for \$1.35? *Ans.* 375.

72. If 75 yards of cloth are worth 65 barrels of flour, and 45 barrels of flour are worth 60 bales of cotton, how many bales of cotton are worth 360 yards of cloth? *Ans.* 416 bales.

73. If \$4=21 francs, 60 francs=7 gold florins of Hanover, 6 gold florins of Hanover=11 gold rubles of Russia, how many gold rubles of Russia are equal to \$360? *Ans.* 404$\frac{1}{4}$.

74. A grocer has 40 pounds of tea worth 50 cents per pound, 75 pounds worth 40 cents per pound, and 25 pounds

worth 34 cents per pound. If he mixes these three kinds of tea, what will a pound of the mixture be worth?

Ans. \$0.41$\frac{1}{14}$.

75. A wine merchant mixed 40 gallons at 48 cents, 80 gallons at 75 cents, and 75 gallons at 60 cents. What was a gallon of the mixture worth? *Ans.* \$0.631.

76. A merchant mixed 50 gallons of wine worth \$1.25 per gallon with 20 gallons of water of no value: what was a gallon of the mixture worth? *Ans.* \$0.89$\frac{2}{7}$.

77. A grocer bought 24 gallons of syrup at 40 cents per gallon, and 48 gallons at 35 cents per gallon. He mixed both quantities of syrup, and 16 gallons of water together, and sold the mixture so as to gain 33$\frac{1}{3}$ per cent. At what price did he sell it per gallon? *Ans.* \$0.40.

78. A person bought 4 dozen eggs at 18$\frac{3}{4}$ cents per dozen, 12 dozen at 16 cents per dozen, and 36 dozen at 14 cents per dozen. How must he sell his eggs per dozen so that he may gain 25 per cent.? *Ans.* \$0.185.

79. On a certain day the mercury in the barometer was observed to stand 4 hours at 63 degrees, 3 hours at 58 degrees, and 5 hours at 55 degrees. What was the mean temperature for these 12 hours? *Ans.* 58$\frac{5}{12}$ degrees.

80. A person bought 1500 bushels of wheat at \$0.87$\frac{1}{2}$ per bushel, and 1600 bushels at 0.93$\frac{3}{4}$ per bushel. At what price per bushel must he sell the whole, in order that he may gain 2 per cent., if he allows 3 per cent. for waste?

Ans. \$0.954, nearly.

81. A farmer has corn worth 60 cents per bushel, and barley worth 45 cents per bushel. How many bushels

of each must he take to form a mixture worth 50 cents per bushel? *Ans.* 5 of corn, and 10 of barley.

82. How much wine, at $1.25 per gallon, and water of no value, must be mixed to form a mixture worth 90 cents per gallon? *Ans.* 90 of wine, and 35 of water.

83. A merchant has one kind of tea worth 80 cents per pound, and another kind worth 45 cents per pound. How many pounds of each must he use to form a mixture worth 60 cents per pound?

Ans. 15 lbs. at 80 cents, and 20 lbs. at 45 cents.

84. A grocer has four kinds of sugar which are worth 8, 9, 11, and 12 cents per pound, respectively. How many pounds of each kind must he use in forming a mixture that is worth 10 cents per pound?

Ans. 2 lbs. of the 8 and 12 cent sugars, and 1 lb. of each of the other kinds.*

85. How many gallons of water of no value must be mixed with 60 gallons of milk worth 4 cents per quart, in order that the mixture may be worth 3 cents per quart? *Ans.* 20 gal.

86. A milkman purchased 80 gallons of milk, at 12 cents per gallon; how many gallons of water must he mix with his milk, so that by selling the mixture at 3 cents per quart he may gain 15 per cent. on the cost of the milk?

Ans. 12 gallons.

87. A grocer has four kinds of tea which are worth 40, 50, 65, and 90 cents per pound, respectively. How many pounds

* NOTE.—We may also find other proportional quantities, which will satisfy the conditions of the question.

of each must he use in forming a mixture of 260 pounds worth 60 cents per pound?

Ans. 120 lbs. of the first, 20 lbs. of the second, 40 lbs. of the third, and 80 lbs. of the fourth kind.

88. A wine merchant has 48 gallons of wine worth \$1.20 cents per gallon, and 32 gallons worth \$0.70 per gallon. He wishes to mix these two quantities of wine with two other kinds, one at \$1.25, the other at \$1.40 gallon. How many gallons of each of the last two kinds must he use, in order that the mixture may be worth \$1.12 per gallon? *Ans.*

89. A person owes \$300, payable in 2 months, \$400 payable in 6 months, and \$800 payable in 9 months. What is the proper time of credit for the payment of the whole sum at once? *Ans.* $6\frac{4}{5}$ months.*

90. A money dealer has due to him a certain sum of money, $\frac{1}{3}$ of which is due in 3 months, $\frac{1}{2}$ in 4 months, and $\frac{1}{6}$ in 6 months. What was the equated time for the payment of the whole? *Ans.* 4 months.

91. A merchant has due to him \$400, payable in 200 days, \$475 payable in 60 days, and 725 dollars payable in 100 days. What is the equated time for the payment of the whole at once? *Ans.* $113\frac{1}{8}$ days.

92. A merchant purchased goods to the amount of \$700. He agreed to pay \$175 in 1 month, \$175 in 2 months, \$175 in 3 months, and \$175 in 8 months. What is the equated time for the payment of the whole? *Ans.* $3\frac{1}{2}$ months.

* NOTE.—The answer to this and those to other examples in Equations of Payments, are obtained by using the rule given in the note to example 15, page 197.

93. A debt of \$1200 is due in 8 months. If the debtor pays \$450 at present, what is the proper time for the payment of the remainder? *Ans.* $12\frac{4}{5}$ months.

94. A lent B \$460 for 8 months, and at another time, he lent him \$240 for 3 months. How long ought B to lend A \$600 to balance the favor? *Ans.* $7\frac{1}{3}$ months.

95. A merchant failing in business finds that he has \$2400, and that his debts amount to \$3120. How much ought he to pay A, whose claim is \$420. *Ans.* \$323.07.

96. Three masons, A, B, and C, were engaged to do a certain work for \$360. A worked 48 days of 12 hours each; B 60 days of 10 hours each; and C 35 days of 8 hours each. How many dollars should each receive? *Ans.*

97. Two merchants, A and B, formed a partnership. A put into trade, at first, \$1600, and at the end of 4 months put in \$400 more; B put into trade \$2400, and at the end of 8 months, he took out \$800 dollars. At the end of a year, they find that they have gained \$1800. How much of the gain belongs to each? *Ans.* A \$840; B \$960.

98. A widow, according to the will of her deceased husband, is required to divide a sum of \$7500 with her two sons and three daughters, so that each son may receive twice as much as each daughter, and the widow \$500 more than all her children together. What was her share, and what was the share of each child? *Ans.* Son \$1000; daughter \$500; widow \$4000.

99. A man sold a horse for \$131.25, and thereby gained 75 per cent. on the cost of the horse. What did the horse cost? *Ans.* \$75.

100. In a certain school $\frac{1}{4}$ of the boys learn to read, $\frac{1}{8}$ learn to write, $\frac{1}{3}$ study arithmetic, $\frac{1}{12}$ study geography, and the re-

mainder, which is 10, study grammar. How many scholars are there in the school? *Ans.* 48.

101. A gentleman expends the third part of his income for board and lodging, the eighth part for clothes and washing, and he saves $338 yearly. What is the amount of his yearly income? *Ans.* $624.

102. A New York merchant wishes to pay a debt of £1200 in London. How many dollars must he pay to procure remittances through France and Hamburg, if we allow that 21 francs =$4, 19 marcs banco at Hamburg equal 35 francs at Paris, and £7 at London, equal 96 marcs banco at Hamburg?

Ans. $5774.43.

103. If £2 and $\frac{5}{8}$ of $\frac{1}{3}$ of a pound will purchase 3 yards and $\frac{2}{3}$ of $\frac{3}{5}$ of a yard; how much may be purchased for 9 shillings and $\frac{5}{8}$ of a shilling? *Ans.* $\frac{3}{4}$ of a yard.

104. Divide $4800 among 3 men, 15 women, and 20 boys, and give each woman $\frac{3}{5}$ of a man's share, and each boy $\frac{2}{3}$ of a woman's share.

Ans. { A man's share is $240.
A boy's share is $96.
A woman's share is $144.

105. A young hare starts 40 yards before a greyhound, and is not perceived by him till she has been up 40 seconds; she scuds away at the rate of 10 miles per hour, and the dog makes after her at the rate of 18 miles per hour. How long will the course hold, and what ground will be run over, counting from the outsetting of the dog?

Ans. $60\frac{5}{22}$ sec., and 530 yards run.

106. A regiment of 800 men is to be clothed, each suit is to contain $3\frac{3}{4}$ yards of cloth that is $1\frac{1}{8}$ yards wide, and lined with serge that is $\frac{5}{8}$ of a yard wide; how many yards of serge will be required to line them? *Ans.* 5400 yards.

107. Suppose 2000 soldiers had been supplied with bread for 12 weeks, allowing each man 14 ounces per day; but they found that 105 barrels, each containing 200 lbs., were spoiled; what must be the daily allowance to each man that the remainder may last the same time? *Ans.* 12 ounces per day.

108. Suppose that I have $\frac{3}{16}$ of a ship, whose whole value is \$3600; what part of her have I left after selling $\frac{2}{5}$ of $\frac{4}{5}$ of my share, and what is this part worth?

Ans. $\frac{37}{240}$; worth \$555.

109. A workman was hired for 36 days, at \$0.75 per day for every day that he worked; but for every day he was idle he was to forfeit \$0.25 How many days did he work when the balance due to him at the end of the time, was \$11.00; and how many days was he idle when he had to receive only one day's wages?

Ans. 25 days, in the former case; and $26\frac{1}{4}$ days in the latter case.

110. A father left his son a fortune, $\frac{1}{4}$ of which he spent in 8 months; $\frac{3}{7}$ of the remainder lasted him 12 months longer, when he had \$820 left. What sum did the father bequeath to his son? *Ans.* $\$1913.33\frac{1}{3}$.

111. If 1000 men, besieged in a town, with provisions for 5 weeks, allowing each man 16 ounces per day, be reinforced with 500 men more; and supposing that they cannot be relieved till the end of 8 weeks; what must be each man's daily allowance, that the provision may last that time?

Ans. $6\frac{2}{3}$ ounces.

112. A father divided his fortune among his three sons A, B, and C, giving A \$4 as often B \$3, and C \$5 as often as B

$6 ; what was the whole legacy, supposing that A's share was $4000 *Ans.* $9500.

113. At what time between 12 and 1 o'clock, do the hour and minute hands of a watch point in directions exactly opposite? *Ans.* $32\frac{8}{11}$ minutes past 1 o'clock.

114. A man and his wife found that when they were together, a bushel of corn would last them 15 days; but when the man was absent, it would last the woman alone 27 days. How long would it last the man alone? *Ans.* $33\frac{3}{4}$ days.

115. If a family of 9 persons spend $450 in 4 months, how many dollars will maintain them 16 months, if 4 persons more be added to the family? *Ans.* $2600.

116. A stationer sold steel pens at $3.60 per thousand, but the demand for the pens having increased, he raised the price to $4.80 per thousand. What did he gain per cent. by the last sale? *Ans.* $33\frac{1}{3}$ per cent.

117. A speculator purchased a village lot for $1200, and sold it at a loss of $12\frac{1}{2}$ per cent., and then purchased another lot with the proceeds of the sale, and sold it at an advance on the cost of 14 per cent. Did he gain or lose by these transaction, and how much? *Ans.* He lost $3.

118. A grocer has sugars worth $12, $11, and $8 per hundred weight, and he wishes to form a mixture containing 35 hundred weight from these three kinds that may be worth $9 per hundred weight; what quantity of each must he take?

Ans. 5 cwt. of each of the first two kinds, and 25 cwt. of the other kind.

119. What is the interest on $450 for 117 days, at 7 per cent., if we allow that the year contains 365 days?

Ans. $10.077*

120. What is the interest on $221 for 335 days, at 7 per cent., if we allow that the year contains 365 days? *Ans.*

121. A person dies worth $10000, and leaves $\frac{1}{3}$ of his property to his wife, $\frac{1}{2}$ to his son, and the rest to his daugh-

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* NOTE.—To find the interest at 10 per cent., let us observe that three times the number of days in the year, or 3×365, equals $=1095$. If this product had been 1000, the interest on one dollar for any given number of days might be found by multiplying the number of days by 3, and forming a decimal of four places from this product by prefixing, if necessary, ciphers to its right. But $1095-1000=95$, and $95\div1095=0.0867$. Therefore, 1095 diminished by 0.0867 of itself, or multiplied by $1-0.0867=0.913$, will be 1000, nearly. Hence, to find the interest at 10 per cent. for any number of days, *multiply the given number of days by* 3, *and from the product form a decimal of four places by prefixing ciphers to its right, if necessary. Then multiply the principal by this decimal, and this last product multiplied by* 0.913 *will give the interest required.*

Having found the interest at 10 per cent., we can obtain the interest at 1 per cent. by removing the decimal point one place towards the left. Knowing the interest at 1 per cent., it is easy to find the interest at any particular rate per cent. For the solution of the 119th question by this method, we have the following operation:

$117\times3=351$; hence the multiplier is 0.0351.

$450\times0.0351=$15.795, and $15.795\times0.913=$14.420835.

Hence, the interest at 1 per cent. is $1.442, nearly.

Therefore, the interest at 7 per cent. is $1.442\times7=$10.094.

It may be seen that this result differs from the true result by nearly 2 cents.

ter. The wife at her death leaves $\frac{3}{5}$ of her legacy to the son, and the rest to the daughter; but the son adds his fortune to his sister's, and gives her $\frac{1}{3}$ of the whole. How much will the sister gain by this arrangement, and what will her gain be of the whole? *Ans.* \$$333\frac{1}{3}$; $\frac{1}{36}$.

122. If 8 men can dig a trench 100 feet long, 3 feet wide, and 4 feet 6 inches deep in 9 days, how many will be required to dig a trench 80 feet long, 5 feet wide, and 2 feet deep, in $5\frac{1}{3}$ days? *Ans.* 8 men.

123. If 3000 copies of a book of 11 sheets require 66 reams of paper, how much paper will be required for 5000 copies of a book of $12\frac{1}{2}$ sheets? *Ans.* 125 reams.

124. If a ton of turnips will last 25 sheep a fortnight, how much will be required to supply 40 sheep during the months of January and February in leap year? *Ans.* $6\frac{6}{7}$ tons.

125. If 7 masons can erect a certain piece of wall in $20\frac{5}{8}$ days of $9\frac{3}{5}$ hours each, how long would it take 3 masons to do $2\frac{3}{4}$ of the same work, reckoning 12 hours to the day?

Ans. $105\frac{7}{8}$ days.

126. A merchant employs \$700 in trade, and at the end of 3 years takes another into partnership, who advances \$1900. At the end of 4 years from this time they have gained \$500; how ought this to be divided between them?

Ans. \$196, \$304.

127. A father left to the elder of his two sons $\frac{13}{25}$ of his estate, and $\frac{13}{25}$ of the remainder to the younger, and the residue to his widow; find their respective legacies, it being found that the elder son received \$1690 more than the younger.

Ans. \$3250, \$1560, \$1440.

128. A and B entered into partnership. A put into the stock

at first \$2000, and at the end of 8 months \$1000 more; B put in at first \$750, and at the end of 4 months \$3000 more, but took out at the end of 3 months \$1300. At the end of the year they find that they have gained \$1635; what should each receive? *Ans.* \$795, \$840.

129. A cistern has two pipes, by one of which it may be filled in 40 minutes, and by the other in 50 minutes; it has also a discharging pipe by which it may be emptied in 25 minutes. If all these three were open at the same time, in what time would the cistern be filled?

Ans. 3 hours and 20 minutes.

130. A, B, and C are sent to empty a cistern by means of two pumps of the same bore; A and B go to work first, A making 37 strokes a minute, and B 40 strokes a minute; but after 5 minutes each makes 5 strokes less a minute, and after 10 minutes more, A gives way to C, who works at the rate of 30 strokes a minute; the cistern is emptied in 32 minutes altogether, and the men are paid \$3.02; what should each receive?

Ans. A \$1.01, B \$1.59, C \$0.42.

131. A piece of work can be done in a day of $11\frac{1}{2}$ hours by 2 men, or 5 women, or 12 boys; in what time could it be done by 1 man, 2 women, and 3 boys? *Ans.* 10 hours.

CHAPTER XI.

REDUCTION OF CURRENCIES.

91. By the term currency is here meant that which circulates as the medium of trade; as specie, bank bills. The *Reduction of currencies* consists in changing any amount of money expressed in the denominations of one country into an equivaalent amount expressed in the denominations of any other country.

92. Foreign coins have three values, an *intrinsic* value, a *commercial value*, and a *legal* value. The *intrinsic* value of a coin is determined by its weight and purity; its *commercial* value is the price which it will bring in market; and its *legal* value is that which is fixed by law.

For example, the intrinsic value of the pound sterling, which is represented by the gold sovereign, is $4.861. The intrinsic value of the sovereign is found by comparing it with our gold eagle. Its commercial value varies from $4.83 to $4.86. When the value of our imports exceeds the value of our exports, and we are consequently obliged to transport specie to pay our foreign debt, the sovereign will command a higher price than it does when we have no foreign debt. The legal or custom house value of the sovereign is $4.84, as fixed by act of Congress in 1842.

93. Since the commercial value of the pound is variable, and

not often the same as the legal value, it follows that we cannot always determine the cost of a *draft* or *bill of exchange** on London, by multiplying the legal value of the pound ($4.84,) by the number of pounds for which the bill is drawn. Merchants, therefore, adopt a different method of finding the cost of a bill of exchange on London.

By an Act of Congress, in 1799, the legal value of the pound was fixed at $\$4.44\frac{4}{9}$, and this value, which is called the *par value*, is made the basis of exchange between this country and England, although it is not now the legal value. To find the commercial value of the pound, the *par value* is increased by a variable percentage of itself. This percentage, which is called the *premium* of exchange, ranges from 8 to 10 per cent.

If the premium of exchange is at 9 per cent., the commercial value of the pound is $\$4.44\frac{4}{9} \times 1.09 = \$4.844$, nearly.

94. Before the adoption of Federal money, all accounts were kept in the currency of Great Britain, and the English denominations of shillings and pence are still used to some extent. These denominations, however, have not the same value in all the States. For, example, the shilling in the New England States is equal to $16\frac{2}{3}$ cents, *or the sixth* of a dollar, while in New York, it is equal to $12\frac{1}{2}$ cents, or the *eighth* of a dollar.

* NOTE.—*A Bill of Exchange* is a written order drawn on a person in a distant place, directing him to pay money to some person mentioned in the bill, or to his order. The person who draws the bill is called the *drawer;* the person who is directed to pay the money is called the *drawee;* the person to whom the money is directed to be paid is called the *payee.* The drawee is also called the *acceptor*, when he has accepted the bill, or obligated himself to pay it. The drawee accepts a bill by writing the word "accepted" across the face or back of the bill, and his name under it.

19*

Nine pence, or $\frac{3}{4}$ of a shilling, in New England, are equal to $12\frac{1}{2}$ cents, but in New-York, 9 pence are equal to $9\frac{3}{8}$ cents. In Pennsylvania, the shilling is equal to $\frac{2}{15}$ of a dollar, or $13\frac{1}{3}$ cents.

The reason that the shilling has not the same value in all the States is owing to the fact that, previous to the adoption of the Constitution and Federal Money, several of the colonies issued Bills of Credit, and these Bills of Credit, which were the colonial currency, depreciated in value, and in some colonies more than in others. In New England the value of the pound in this currency was $\$3.33\frac{1}{3}$. Hence, 6 shillings were worth $\frac{6}{20}$ of $\$3.33\frac{1}{3}=\$1$. In New York the pound was worth \$2.50. Hence, 8 shillings were worth $\frac{8}{20}$ of $\$2.50=\$1$.*

95. The value in Federal money of the shilling of the old currencies of the several States, is given in the following

TABLE.

	s.	*cts.*	*s.*	\$
In New England, Va., Ky., and Tenn.,	1	$=16\frac{2}{3}$ ∴	6	=1.
In New York, Ohio, and N. Carolina,	1	$=12\frac{1}{2}$ ∴	8	=1.
In New Jersey, Penn., Del., and Md.,	1	$=13\frac{1}{3}$ ∴	$7\frac{1}{2}$	=1.
In South Carolina and Georgia, . .	1	$=21\frac{3}{7}$ ∴	$4\frac{2}{3}$	=1.

EXAMPLES.

1. A New York merchant owes £380 15s. in London, and he wishes to purchase a bill of exchange on London, which will

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* NOTE.—There is no good reason for observing these arbitrary divisions of the dollar. Since the currency of the United States is universally acknowledged to be the best currency in the world, all merchandise should be sold at prices stated in Federal money.

cancel his debt. How much must he pay for his bill, exchange being at 9 per cent. premium?

OPERATION.

$$£380\ 15s. = £380.75.$$
$$\$4.44\tfrac{4}{9} \times 380.75 = \$\tfrac{40}{9} \times 380.75 = \$1692.22\tfrac{2}{9}.$$
$$\$1692.22\tfrac{2}{9} \times 1.09 = \$1844.52\tfrac{2}{9}.\ \textit{Ans.}$$

Explanation.—Since the old par value of \$1 is $\$4.44\frac{4}{9} = \$\frac{40}{9}$, the par value of £380.75. is 380.75 times $\$\frac{40}{9} = \$1692.22\frac{2}{9}$. This sum increased by 9 per cent. of itself is $\$1844.52\frac{2}{9}$.

2. Reduce £145 12s. New York currency to Federal money. *Ans.* \$364.

3. Reduce £84 16s. Pennsylvania currency to Federal money. *Ans.* $\$226.13\frac{1}{3}$.

4. Reduce £450 7s. 6d. Georgia currency to Federal money. *Ans.* \$1930.18, nearly.

5. Reduce £720 18s. 9d. New England currency to Federal money. *Ans.* $\$2403.12\frac{1}{2}$.

6. Reduce £480 4s. 9d. New York currency to Georgia currency. *Ans.* £280 2s. 9d.

7. Reduce \$385.45 to New England currency. *Ans.* £115 12s. 8d.

8. Reduce \$1200.80 to Pennsylvania currency. *Ans.* £470 6s.

9. Reduce \$2485 to New York currency. *Ans.* £994.

10. A merchant in Boston owing £640 in Liverpool, wishes to purchase a bill of exchange on Liverpool that will cancel the

debt. How much must he pay, exchange being at $9\frac{1}{2}$ per cent. premium? *Ans.* \$3114$\frac{2}{3}$.

11. What will it cost to purchase a bill on Paris for 2400 francs, at 1 per cent. above par, the par value being \$0.186 per franc? *Ans.* \$491.04.

12. At $8\frac{1}{2}$ per cent. premium, what will be the amount of a bill on London which I can purchase for \$6510? *Ans* £1350.

CHAPTER XII.

RATIO AND PROPORTION.

DEFINITIONS.

1. *Ratio* is the quotient which arises from dividing one number, or quantity, by another of the same denomination.* Thus, the ratio of 12 to 6 is $6 \div 12 = \frac{1}{2}$. The ratio of 8 to 6 is $6 \div 8 = \frac{6}{8} = \frac{3}{4}$.

2. Of the two numbers which form a ratio, the first is called the *antecedent*, and the second is called the *consequent*. A ratio is expressed by writing two dots between the antecedent and consequent. Thus the ratio of 16 to 18 is expressed 16 : 18. A ratio may also be expressed in the form of a fraction by making the antecedent the denominator, and the consequent the numerator of the fraction.

3. The ratio of one number, taken as an antecedent, to another, taken as a consequent, is called a *Simple Ratio*.

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* NOTE.—There can be no ratio between quantities of different denominations, since, for example, it would be absurd to ask how many times 1 dollar is contained in 12 yards. When two denominate numbers can be reduced to the same denomination, their ratio can be found. The ratio of two numbers, or quantities, is always an abstract number.

4. A *Compound Ratio* is the ratio of the *product* of the antecedents to the *product* of the consequents. Thus the compound ratio of 8 : 12 and 6 : 10 is $8\times6 : 12\times10$, or, 48 : 60.

5. If the ratio of two numbers is equal to the ratio of two other numbers, the four numbers taken together, constitute a proportion. Thus, the ratio of 16 to 4 being equal to the ratio of 24 to 6, the four numbers, 16, 4, 24, 6, constitute a proportion, which is written 16 : 4 :: 24 : 6, and it is read 16 is to 4 as 24 is to 6. The four dots placed between 4 and 24 are equivalent to the sign of equality, and this sign is sometimes used in their stead.

6. The first and fourth terms of a proportion are called the *extremes*, and the second and third terms are called the *means.*

96. From the foregoing definitions several consequences may be drawn, among which are the following:

1. *If both terms of a ratio be multiplied or divided by the same number, the value of the ratio will not be changed.*

2. *If the antecedent of a ratio be multiplied, or its consequent be divided, by any number, the value of the ratio will be divided by this number.*

3. *If the antecedent of a ratio be divided, or its consequent be multiplied, by any number, the value of the ratio will be multiplied by this number.*

4. *If the antecedents or consequents of any proportion be multiplied or divided by the same number, the proportion will not be destroyed.*

5. *If the first and second terms, or the first and third terms, of a proportion be multiplied or divided by the same number, the proportion will not be destroyed.*

97. In any proportion, *the product of the extremes is equal to the product of the means.** Take the proportion, 8 : 12 :: 4 : 6; then, since the four numbers are in proportion, the ratio of 8 to 12 must equal the ratio of 4 to 6; that is, $\frac{12}{8}=\frac{6}{4}$. If we multiply each of these equal ratios by 8 times 4, we shall obtain $\frac{8\times 12\times 4}{8}=\frac{4\times 8\times 6}{4}$, or, by cancelling the factors 8 and 4, we have $8\times 6=4\times 12$; that is, the product of the extremes is equal to the product of the means. Hence, if the first three terms of a proportion are given, the fourth term may be found by dividing the product of the second and third terms by the first term.

98. All questions, or problems, in which three terms are given, and it is required to find a fourth term, such that the ratio of one of the given terms to this fourth term shall be the same as the ratio of the other two given terms, may be solved by proportion. We will solve by proportion the following question:

If 8 yards of cloth cost \$24, what will be the cost of 9 yards at the same rate?

Here it is plain that 8 yards must have the same ratio to 9 yards, that the cost of 8 yards has to the cost of 9 yards. Hence, we may have the following proportion:

8 *yds.* : 9 *yds.* :: \$24 : the cost of 9 yards.

Now, by the last article, the fourth term is equal to

$$\frac{24\times 9}{8}=3\times 9=27;$$

that is, the cost of 9 yards is \$27.

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* NOTE.—The propositions in proportion can only be demonstrated in a *general* manner by the aid of Algebra, and we shall not, therefore, give them a place in this work.

99. From the principles of ratio and proportion, which we have stated and explained, we deduce the following rule for solving questions in proportion.

RULE.

I. *Make that number which is of the same kind as the answer sought, the third term of a proportion.*

II. *Then, if by the nature of the question the answer must be greater than the third term, make the greater of the two remaining numbers the second term, and the less the first term; but if the answer must be less, make the less of the two remaining numbers the second, and the other number the first term.*

III. *Divide the product of the second and third terms by the first term, and the quotient will be the fourth term of the proportion, or the answer sought.*

If the first and second terms are not of the same denomination, they must be reduced to the same denomination before instituting the proportion. When the third term is a denominate number, it is generally better to reduce it to the lowest denomination mentioned in it.

The student should observe to abbreviate the operations in proportion by cancellation, whenever such abbreviation is possible.

EXAMPLES.

1. If 18 acres of land cost \$364.50, what will 175 acres cost at the same rate? *Ans.* \$3543.75.

2. If $\frac{3}{16}$ of an acre of land cost \$51.15, what would a whole acre cost at that rate? *Ans.* \$280.80.

3. If 17 hats cost \$68, what would 45 hats cost at the same rate? *Ans.* \$180.

4. If 18 men can mow a meadow in 8 days, in how many days can 24 men mow the same meadow? *Ans.* 6.

5. If the interest on \$100 for one year is \$7, what is the interest on \$360 for 2 years and 8 months? *Ans.* \$67.20.

6. If 24 pounds of sugar cost \$2.40, how many pounds can be bought for \$75? *Ans.* 750.

7. A and B hired a pasture for \$24, in which A had 8 cows for 12 weeks, and B 10 cows for the same time. How much must each pay? *Ans.* A \$10$\frac{2}{3}$; B \$13$\frac{1}{3}$.

8. A can chop a cord of wood in 5$\frac{1}{3}$ hours, and B in 4 hours. In what time can they chop 35 cords?

Ans. 80 hours.

9. If an upright pole that is 12 feet long casts a shadow of 16 feet, how high is that steeple, the length of whose shadow is 154 feet? *Ans.* 115$\frac{1}{2}$ feet.

10. If $\frac{3}{4}$ of a yard of cloth cost \$3.75, what will 18 yards cost at the same rate? *Ans.* \$90.

11. If it cost \$480 to pave a road 160 rods long, what will it cost to pave a road that is 10 miles, 3 furlongs and 25 rods long, at the same rate? *Ans.* \$10035.

12. If it requires 12 yards of cloth that is $\frac{3}{4}$ of a yard wide to make a cloak, how many yards of cloth that is $\frac{7}{8}$ of a yard wide, will it require to make the cloak? *Ans.* 10$\frac{2}{7}$.

13. If 45 bags will contain 32 bushels of grain, how many bags one-half the size will contain 480 bushels? *Ans.* 1350.

14. Twelve masons can do a certain work in 24 days. After they have worked 14 days, they wish to complete the work in 4 more days. How many additional workmen must be employed. *Ans.* 18.

15. If $3\frac{7}{8}$ pounds of tea cost $\$2.90\frac{5}{8}$, what is the cost of 24 pounds and 12 ounces at the same rate? *Ans.* $\$18\frac{9}{16}$.

16. A farmer paid $2000 for 150 acres; at that rate how much should he pay for 15 acres, 2 roods and 20 rods?

Ans. $\$208\frac{1}{3}$.

17. Eighteen men can mow a field in 24 days. They work together for 8 days, and then 8 men quit work, leaving the remaining men to finish the field. In what time will the field be mowed? *Ans.* $28\frac{4}{5}$ days.

18. If 18 bushels of clover seed cost $144, what is the cost of 12 bushels and 6 quarts? *Ans.* $97.50.

19. If 12 pounds of sugar cost $1.68, and 16 pounds of sugar are worth 14 pounds of coffee, what is the cost of 240 pounds of coffee? *Ans.* $38.40.

20. A cistern which holds 2400 gallons is supplied with water by a pipe which lets into the cistern 8 gallons per minute. By leakage 3 gallons run out per minute. Now if the leakage is stopped when the cistern is half full, in how many hours will the cistern be full? *Ans.* 390.

21. A besieged town, containing 22400 inhabitants, has provisions to last 3 weeks; how many must be sent away that the provisions may last them 7 weeks? *Ans.* 12800.

22. What is the height of a steeple, whose shadow was 148 feet and 4 inches at the same time that the shadow of a staff 6 feet and 8 inches long, was $6\frac{1}{4}$ feet?

Ans. 178 ft. $11\frac{1}{3}$ in.

23. A coach goes from Homer to Syracuse at the rate of 5 miles an hour in 6 hours; in what time would the distance be performed on the railroad at the rate of 20 miles an hour?

Ans. $1\frac{1}{2}$ hours.

24. A borrowed of B \$175.25 for 102 days, and afterwards would return the favor by lending B the sum of \$210.30; how long should he lend it? *Ans.* 85 days.

25. If $2\frac{1}{4}$ yards cost \$12, what will be the cost of $2\frac{5}{8}$ yards? *Ans.* \$14.

26. A grocer bought 27 bushels of apples at 25 cents per bushel, and three times as many bushels of potatoes. He paid for the apples and potatoes \$31.05. What did he pay per bushel for the potatoes? *Ans.* 30 cents.

COMPOUND PROPORTION.

100. *A Compound Proportion* is one in which a compound ratio is equal to a simple one. Thus

$$\left.\begin{array}{l} 3 : 4 \\ 6 : 9 \end{array}\right\} :: 8 : 16$$

is a compound proportion, since

$$3 \times 6 : 4 \times 9 :: 8 : 16;$$

$$\text{or} \quad 18 : 36 :: 8 : 16.$$

Compound Proportion may be applied to the solution of questions which might be solved by the rule given in Simple Proportion, by making two or more statements. For the purpose of discovering some rule for solving such questions, we propose the following problem:

If a man can walk 400 miles in 10 days, by walking 12 hours per day, how many miles can he walk in 25 days, by walking 9 hours per day?

It is obvious that if we find the number of *hours* which he

travels in each case, the question may then be solved by the rule given in Simple Proportion, since it will then involve only *three* terms. Now, in the first case he travels $\overline{10\times12}$ hours, and in the second case he travels $\overline{25\times9}$ hours. Since $\overline{25\times9}$ is greater than $\overline{12\times10}$, it follows that the answer to the question must be greater than 400. Hence we have the following proportion,

$$\overline{10\times12}\ \textit{hours} :: \overline{25\times9}\ \textit{hours} :: \overline{400}\text{ miles} : \textit{Answer}.$$

But, by the definition of a compound ratio, the ratio of 10×12 to 25×9, is equal to the compound ratio of the two simple ratios, 10 : 25 and 12 : 9. Hence, the above proportion is equivalent to the following compound proportion :

$$\left.\begin{matrix} 10\,da. & : & 25\,da \\ 12\,h. & : & 9\,h. \end{matrix}\right\} :: 400\text{ miles} : \textit{Answer}.$$

The fourth term of this proposition, or the answer sought, is

$$\frac{\overset{10}{\cancel{400}}\times25\times\overset{3}{\cancel{9}}}{\cancel{10}\times\underset{3}{\cancel{12}}} = 750.^*$$

---

* Note.—In this question, if we suppose, at first, that the days are of the *same* length, we can find the distance which the man woul.' travel in 25 days, by walking 12 hours per day, by the following sim ple proportion :

da.		*da.*		*miles.*		*miles.*
10	:	25	: :	400	:	x, the number of days required.

Having found x, we have the following proportion to find the distance he would travel, by walking 9 hours per day :

hours.		*hours.*		*days.*		*days.*
12	:	9	: :	x	:	y, the number of days required.

From the foregoing explanation and principles we derive the following rule for the solution of questions in compound proportion.

RULE.

I. *Place that number which is of the same kind as the answer required for the third term of the proportion.*

II. *Then, of the remaining numbers, take each two that are of the same kind, and arrange them as in simple proportion.*

III. *Divide the product of the numbers in the second and third terms by the product of the numbers in the first term, and the quotient will be the fourth term of the proportion, or the answer sought.*

The solution of the question, then, involves these two proportions:

$$10 : 25 :: 400 : x, \qquad (1)$$
and
$$12 : 9 :: x : y. \qquad (2)$$

From (1), $$\frac{25}{10} = \frac{x}{100},$$

From (2), $$\frac{9}{12} = \frac{y}{x}$$

Now, if equals be multiplied by equals, the products are equal; we may therefore have

$$\frac{25\times 9}{10\times 12} = \frac{x\times y}{400\times x} \qquad \text{(A.)}$$

or, by cancelling x, $$\frac{25\times 9}{10\times 12} = \frac{y}{400}$$

Whence, multiplying by 400, $y = \dfrac{25\times 9\times 400}{10\times 12} = 750.$

From (A.) we see that the products of the corresponding terms in two proportions are proportionals.

It must be observed, in forming the first and second terms of the proportion, that each antecedent must be of the same denomination as its consequent.

EXAMPLES.

1. If 18 men can chop 144 cords of wood in 6 days, by working 10 hours per day, how many cords can 24 men chop in 21 days, by working 12 hours per day? *Ans.* 807$\frac{2}{5}$.

2. If 48 pounds of wool will make 54 yards of cloth that is 5 quarters wide, how many yards of the same kind of cloth that is 3 quarters wide, will 288 pounds make? *Ans.* 540.

3. If it require 96 bushels of wheat to sow a piece of ground that is 64 rods wide and 80 rods long, how many bushels of wheat will it require to sow a piece of ground that is 192 rods long and 128 rods wide? *Ans.* 460$\frac{4}{5}$.

4. If the interest on \$100 for 18 months is \$9, what is the interest on \$250 for 8 months? *Ans.* \$10.

5. If 18 horses consume 54 bushels in 6 days, how many bushels will 72 horses consume in 63 days? *Ans.* 2268.

6. If 2 pounds of thread make 6 yards of linen of 1$\frac{1}{4}$ yards wide, how many pounds of thread will be required to make 45 yards of linen that is 1 yard wide? *Ans.* 12 pounds.

7. There are in a fortress 80000 *cwt.* of ammunition, which must be removed in 9 days. It is found that in 6 days 18 horses have carried away 4500 *cwt.* How many horses will be required to carry away the remainder in 3 days?

Ans. 640.

8. It is found that 5 compositors can compose, in 16 days, by working 14 hours per day, 20 sheets of 24 pages in each sheet, 50 lines in a page, and 40 letters in a line. At this

rate, in how many days that are 7 hours long, can 10 compositors compose a volume containing 40 sheets, 16 pages in a sheet, 60 lines in a page, and 50 letters in a line?

Ans. 32 days.

9. If 6 shoemakers make, in 4 weeks, 36 pair of men's, and 24 pair of women's shoes, how many pair of each kind can 18 shoemakers make in 5 weeks?

Ans. 135 pair of men's, and 90 pair of women's shoes.

10. A wall is to be built 27 feet high, and 9 feet of it are built by 12 men in 6 days; how many men must be employed to finish the remainder in 4 days? *Ans.* 36.

11. If 12 horses draw 44 tons of stones in 5 days, how many horses can draw 132 tons the same distance in 18 days?

Ans. 10.

12. If a footman travels 130 miles in 3 days, when the days are 14 hours long, in how many days, of 7 hours each, can he travel 390 miles? *Ans.* 18.

13. If \$450 gain \$18 in 8 months, at 6 per cent., in what time will \$360 gain \$33.60, at 7 per cent.? *Ans.* 16 months.

14. If a garrison of 800 men have provisions which will last them 12 weeks, each man being allowed 14 ounces per day, how many men will the same provisions last 16 weeks, if each man is allowed 8 ounces per day? *Ans.* 1050.

15. If a stone which is 12 feet long, 8 feet wide, and 4 feet thick, weighs 11520 pounds, what will one of the same kind weigh that is 10 feet long, 6 feet wide, and 3 feet thick?

Ans. 5400 pounds.

16. If 18 men can dig a trench 360 feet long, 3 feet deep, and 4 feet wide, in 36 days, how many men will be required to

dig one 400 feet long, 6 feet deep, and 3 feet wide, in 27 days? *Ans.* 40.

17. If the wages of 24 men amount to \$4000, in 8 months, what will the wages of 15 men amount to, in 12 months? *Ans.* \$3750.

18. If 6 painters can paint a building in 24 days, by working 9 hours per day, how many painters can paint the same building in 18 days, by working 12 hours per day? *Ans.* 6.

19. A person completes a journey of 160 miles in 3 days, travelling 11 hours per day; in how many days would he complete a journey of 1000 miles, going 15 hours a day, at the same rate? *Ans.* $13\frac{3}{4}$.

20. If 7 masons can erect a certain piece of wall in $20\frac{5}{8}$ days of $9\frac{3}{5}$ hours each, how long would it take 3 masons to do $2\frac{3}{4}$ of the same work, reckoning 12 hours to the day? *Ans.* $105\frac{7}{8}$ days.

21. If 6 iron bars 4 feet long, 3 inches wide, and 2 inches thick, weigh 288 pounds, how much will 15 bars weigh, each being $6\frac{1}{2}$ feet long, 4 inches wide, and 3 inches thick? *Ans.* 2340 pounds.

22. If a ton of turnips will last 25 sheep a fortnight, how much will be required to supply 40 sheep during the months of January and February, in Leap years? *Ans.*

23. If 20 men can perform a piece of work in 12 days, how many men can perform another piece of work, 3 times as large, in a fifth part of the time? *Ans.* 300.

24. If 3000 copies of a book of 11 sheets require 66 reams of paper, how much paper will be required for 5000 copies of a book of $12\frac{1}{2}$ sheets? *Ans.* 125 reams.

CHAPTER XIII.

DUODECIMALS.

101. *Duodecimals* are a kind of fractions whose denominators are 12, or some power* of 12. Hence the denominations of duodecimals increase or decrease in a *twelvefold* ratio. We may, therefore, adopt a method for writing them, which is similar to that which has been used for writing decimals. Thus, $\frac{5}{12}$ may be expressed 0.5, and $\frac{3}{144}$ may be written 0.03, and so on. When they are so written, we must be careful not to confound them with *decimal* fractions.

102. If we adopt the *duodecimal* system of notation, we shall be obliged to employ two more characters in addition to those which have been employed in the *decimal* system; for it is plain that we must have a character, or figure, to represent *ten*, and also one to represent *eleven*, in order that we may express *all* numbers in this system. Let, then, X=10, ‖=11

103. The rules and principles in duodecimal arithmetic, as well as their explanations or demonstrations, are similar to those in decimal arithmetic. We must carefully observe that, in making our computations in this system of notation, *twelve* units of any order make one of the next higher order.

104. To find what number expressed in the *duodecimal*

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* NOTE.—A power of a number is the product which is obtained by using only that number as a factor. The first power of a number is the number itself. See *Involution*.

notation, will represent a number expressed in the *decimal* notation, we have the following

RULE.

*Divide the given number by twelve, and divide the quotient thus found by twelve, and so on, till a quotient is obtained which is less than twelve. To the last quotient annex the several remainders taken in a reverse order, and observe that when there is no remainder, a cipher must occupy its place.**

105. To find what number expressed in the *decimal* notation, will represent a number expressed in the *duodecimal* notation, we have the following

RULE.

Multiply the left hand figure in the given number by twelve, and to the product add the next figure. Multiply this sum by twelve, and to the product add the next figure. Proceed in this manner till each of the figures in the given number has been used. The last result obtained will be the number required.†

EXAMPLES.

1. Multiply 2X3 by 412.

```
      2X3
      412
      ---
      586
     2X3
   || 50
   ---------
   || 83 || 6
```

Explanation.—Here we have multiplied 2 *one hundred and*

---

* Note.—The student will notice that the explanation of this rule is similar to that which has been given of the rule for Reduction Ascending.

† Note.—The explanation of this rule is similar to that given of the rule for Reduction Descending.

forty-fours, X *twelves, and* 3 *units* by 4 *one hundred and forty-fours*, 1 *twelve, and two units.* In multiplying, we say that 2 times 3 (units) are 6 (units), which we set down in the place of units. We then say that 2 times X (twelves) are 20 (twelves), which are equal to 1 *one hundred and forty-four* and 8 twelves. The 8 twelves we set down in the place of twelves. Then we say that 2 times 2 (one hundred and forty-fours) are 4 (one hundred and forty-fours), to which we add the 1 one hundred and forty-four, and obtain 5 (one hundred and forty-fours). The other partial products are obtained in a similar manner.

In finding the sum of the partial products, we must also remember that it takes 12 units of any order to make one of the next higher order, and therefore, when the sum of the numbers in any column is less than 12, we set it down entire, and when it is greater than 12, we must divide it by 12, set down the remainder under the column added, and add the quotient to the sum of the numbers in the next column. In this example we find that the sum of the partial products, or the product demanded is || 83 || 6.

2. Divide || 83 || 6 by 2X3.

OPERATION.

```
2X3) || 83 || 6(412=quotient.
     || 50
     ----
       33 ||
       2X3
       ---
         586
         586
         ---
           0
```

3. Find the sum of X304, 3 || 0X, 264, and 5851.

Ans. 18107.

4. Find the sum of 9846, 3254, 3X0 || and 580.

Ans. 15269.

5. From 532X subtract 4X || 3. *Ans.* 437.

6. Multiply 385 by 2 || . *Ans.* X967.

7. Multiply || 210 by 1X. *Ans.* 1859X0.

8. Multiply 4. || by X.3*. *Ans.* 42.49.

9. Multiply 3.2X by 3.1. *Ans.* 9. || 8X.

10. Divide 710X by 21. *Ans.* 34X.

11. Divide 1076 || by 61. *Ans.* 20 || .

12. Divide 9. || 8X by 3.2X. *Ans.* 3.1.

13. Divide 42.49 by 4. || . *Ans.* X.3.

14. Divide X967 by 2 || . *Ans.* 385.

15. Change 3845 from the decimal notation to an equivalent number expressed in the duodecimal notation. *Ans.* 2285.

16. Change 33X || from the duodecimal notation to an equivalent number expressed in the decimal notation.

Ans. 5747.

17. Change 45 || X from the duodecimal notation to an equivalent number expressed in the decimal notation.

Ans. 7774.

---

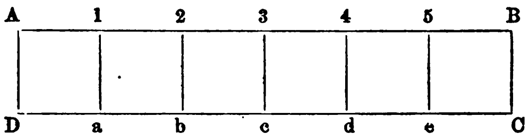
* Note.—In the multiplication and division of duodecimals, we must place the *duodecimal* point according to the directions which have been given in Decimal Fractions for placing the *decimal* point; for *twelfths* multiplied by *twelfths* produce *one hundred and forty-fourths*, and so on. That is, the product of two *duodecimal* fractions must have as many duodecimal places as there are in both factors. Division is the reverse of multiplication; hence, &c.

106. Duodecimals are applied to the measurement of *surfaces* and *solids*. Before pointing out this application, we will give the solutions of the following

PROBLEM I.

*Having given the length and breadth of a rectangular figure, it is required to find its area.**

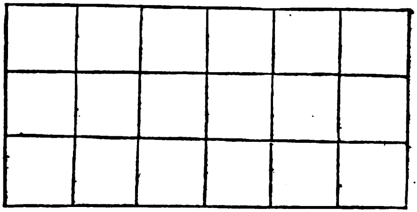
Take a rectangle that is 1 foot wide and 6 feet long. Such a rectangle may be represented by the following figure:



Since the length and breath are expressed in feet, the *measuring unit* of the rectangle must be a square foot. Now, if we divide the line A B into 6 equal parts, at the points 1, 2, 3, 4, 5, and then draw the lines 1*a*, 2*b*, 3*c*, 4*d*, 5*e*, at rightangles to A B, it is then plain that the rectangle A B C D contains the measuring unit 6 times, and its area is therefore 6 square feet.

If the rectangle A B C D had been 3 feet wide instead of 1 foot wide, as represented in the following figure, it is clear that the area would have been 3 times 6 square feet, or 18 square feet, since for each unit in height, or width, there are 6

* NOTE.—A rectangular figure is one having four sides, the opposite sides being equal and its angles rightangles. When one straight line is drawn to a point in another straight line, in such a manner that the two angles formed are equal, these two angles are called right angles.



measuring units, and for 3 units in height, there are 3 times 6 measuring units, or 18 square feet. Hence, we may infer that *the area of any rectangular figure is found by multiplying its length by its width.*

PROBLEM II.

*Having given the length, width, and height of a rectangular solid, it is required to find its solidity.**

Take a rectangular solid which is 5 feet long, 3 feet wide, and 1 foot high. It is plain that we can divide this solid into as many cubic feet as there are square feet in its base; that

is, into $5 \times 3 = 15$ cubic feet. If the height of the solid had

* NOTE.—By the term rectangular solid is meant a solid, the opposite faces of which are rectangles. When the faces are all squares it

been 2 feet, as represented in the figure, then it would have contained twice as many cubic feet; if it had been 3 feet, it would have contained 3 times as many cubic feet, and so on. Hence, *the solidity of a rectangular solid is equal to the product of its length, width, and height.*

107. We will now make an application of Duodecimals in measuring surfaces and solids.

The foot is divided into *twelve* equal parts called *inches* or *primes*, and these are subdivided into twelve other equal parts, called *seconds*, and so on. In expressing *length*, the inch, or prime, is $\frac{1}{12}$ part of a *linear* foot; in expressing *surface*, it is the $\frac{1}{12}$ part of a *square* foot; and in expressing solidity, it is the $\frac{1}{12}$ part of a *cubic* foot. Similar observations may be made in relation to *seconds*, *thirds*, *fourths*, and so on.

TABLE.

12 fourths ('''')	make	1 third,	marked	'''.
12 thirds	"	1 second,	"	''.
12 seconds	"	1 inch or prime	"	*in.* or '.
12 inches or primes	"	1 foot	"	*ft.*

In measuring surfaces and solids, when their dimensions are expressed in feet and duodecimals of a foot, we can first change the numbers expressing feet to equivalent numbers in the duodecimal system of notation, and then proceed according to the directions given in articles 103 and 106;* or we can convert

is called a cube. In measuring solids, the *unit of measure* is a cube. The teacher can give the young pupil a better idea of the solids in geometry by means of wooden figures prepared for the purpose.

* NOTE.—The integral part of the product thus obtained can then be changed to an equivalent number expressed in the decimal system of notation.

the duodecimals of a foot into common fractions, and then proceed as directed in Problem I. and II. The usual method, however, of finding the product of two quantities expressed in feet and duodecimals of a foot, is to multiply each term in the multiplicand by each term in the multiplier, and then find the sum of the partial products for the required product. We shall give a solution of an example by each of these methods.

FIRST METHOD.

How many square feet are there in a floor which is 18 ft. 9 in. long, and 14 ft. 3 in. wide?

Since 12 is contained once in 18, with a remainder of 6, and in 14 once, with a remainder of 2, the duodecimal expressions for 18 ft. 9 in. and 14 ft. 3 in., are 16.9 and 12.3, respectively. For finding the product of 16.9 and 12.3, we have the following

OPERATION.

```
  16.9
  12.3
  ----
   483
  316
 169
 -----
 1X3.23 sq. ft.
```

By Problem I., this product expresses the number of square feet in the floor. For changing the integral part of this product to an equivalent number expressed in the decimal system of notation, we have the following

OPERATION.

```
  1X3    (Rule in Art. 105.)
 12
 --
 22
 12
 ---
267
```

Hence, 1 X 3.23 sq. ft.=267 sq. ft. 2′ 3″= $(267+\frac{2}{12}+\frac{3}{144})$ sq. ft.$=267\frac{3}{16}$ square feet.

SECOND METHOD.

How many square feet are there in a pavement that is 18 ft. 8 in. long, and 5 ft. 3 in. wide?

Here, we observe that 18 ft. 8 in.$=18\frac{8}{12}$ ft.$=18\frac{2}{3}$ ft.;
also, that 5 ft. 3 in.$=5\frac{3}{12}$ ft.$=5\frac{1}{4}$ ft.

Hence, (18 ft. 8 in.) $\times$ (5 ft. 3 in.)$=18\frac{2}{3}\times5\frac{1}{4}=\frac{56}{3}\times\frac{21}{4}=$ 98 ft., the number of square feet in the pavement. The product $\frac{56}{3}$ and $\frac{21}{4}$, is readily obtained by cancelling 4 and 3, which are factors of the numerators and denominators.*

THIRD METHOD.

How many square feet are there in a stone that is 15 ft. 7 in. long, and 8 ft. 8 in. wide?

OPERATION.

$$\begin{array}{rrrr}
 & 15\,ft. & 7' & \\
 & 8\,ft. & 5'\dagger & \\
\hline
 & 124 & 8' & \\
 & 6 & 5' & 11'' \\
\hline
Ans. & 131\,ft. & 1' & 11''
\end{array}$$

* NOTE.—We can also find the number of square feet in the pavement by reducing its length and width to inches, and then finding their product, and dividing it by 144, since 144 square inches make one square foot.

† NOTE.—We call the multiplier 8 ft. 5 in. for the sake of rendering it more convenient to deduce a rule from the above operation for the solution of similar questions. We have already remarked that the multiplier must always be regarded as an *abstract* number. Hence, in giving the *reason* for the above process, we must regard the multiplier as being $8\frac{5}{12}$, an abstract number.

Explanation.—It is evident that a stone that is 15 ft. 7 in. long, and 1 foot wide, contains $15\frac{7}{12}$ square feet or 15 sq. ft. 7′. Hence, a stone that has the same length, and is 8 feet wide, contains 8 times $15\frac{7}{12}$ square feet, or 8 times 8 sq. ft. 7′, which is 120 $\frac{56}{12}$ square feet$=124\frac{8}{12}$ square feet$=$124 sq. ft. 8′.

Again, since a stone that is 15 ft. 7 in. long, and 1 foot wide, contains $15\frac{7}{12}$ square feet$=$15 sq. ft. 7′, one that is 5 in. wide, or $\frac{5}{12}$ of a foot, must contain $\frac{5}{12}$ of $15\frac{7}{12}$ square feet$=\frac{5}{12}\times$(15 sq. ft. 7′)$=(\frac{75}{12}+\frac{35}{144}$*) square feet$=(\frac{77}{12}+\frac{11}{144})$ square feet$=$6 sq. ft. 5′ 11″. If to this number of square feet, we add the number which we before obtained, 124 sq. ft. 8′, we shall obviously obtain the number of square feet in the stone. By making the addition, we find that the stone contains 131 sq. ft. 1′ 11″ By recollecting that, in the product of two dimensions, length and width, the unit is a *square*, and that, in the product of three dimensions, length, width, and height, the unit is a *cube*, we need only employ the symbols, ft., ′, ″, ‴, &c., to denote the several terms of a quantity in which the unit is *linear*, *square*, or *cubic*. Hence, we write the product, 131 sq. ft. 1′, 11″, 131 ft. 1′ 11″, it being understood that the feet are square feet.

From the foregoing operation, we may deduce the following rule for finding the product of two quantities exprsesed in feet, and the duodecimals of a foot.

RULE.

I. *Place the different terms of the multiplier under the corresponding ones of the multiplicand.*

---

* Note.—Observe that the product of 7′ and 5′ is 35″$=$2′ 11″; that is, *the product of any* two terms has as many accents, or *indices*, *as there* are indices in the two terms, taken together.

II. *Multiply the right hand term in the multiplicand, and each of its successive terms, by the left hand term in the multiplier, and to each product annex as many accents (′) as there are in both of its factors. If any product exceeds twelve, divide it by twelve, set down the remainder, and add the quotient to the next product.*

III. *Proceed in a similar manner with each of the remaining terms of the multiplier, and the sum of the several products thus obtained, will be the product required.*

EXAMPLES.

1. What is the product of 14 ft. 2′ and 7 ft. 6′? *Ans.*

2. What is the product of 21 ft. 3′ and 31 ft. 8′?
Ans.

3. What is the product of 25 ft. 1′ 3″ and 41 ft. 3′ 5″?*
Ans.

4. At \$1.87½ per hundred, what is the cost of a board that is 12 feet long and 9 inches wide? *Ans.*

5. At 8 cents per square foot, what will it cost to pave a walk that is 23 ft. 4′ long, and 2 ft. 6 in. wide? *Ans.* \$4.66⅔.

6. How many square yards will it take to carpet a room that is 20 ft. 3 in. long, and 15 ft. 4 in. wide; and what will the carpet cost, at the rate of \$1.50 per square yard?
Ans. 34½ square yds.; cost \$51.75.

7. How much must be paid for the paper to paper the sides of a room that is 20 ft. 8 in. long, 15 ft. 4 in. wide, and 9 feet high, if the paper is worth 31¼ cents per roll, and each roll contains 9 yards of paper that is 18 inches wide? *Ans.* \$5.

8. What will it cost to plaster a room that is 19 ft. 10 in.

---

* Solve this by each of the three methods.

long, 16 ft. 2 in. wide, and 9 feet high, at the rate of $12\frac{1}{2}$ cents per square yard? *Ans.* \$9.

9. How many yards of carpeting that is 18 inches wide, will be required to carpet a floor that is 18 feet long, and 13 ft. 6 in. wide; and what will the carpet cost at \$0.$87\frac{1}{2}$ per yard? *Ans.* 54 yards; cost \$47.25.

10. How many stones, each of which is 2 ft. 3 in. long, and 1 ft. 4 in. wide, will be required to pave a walk that is 13 ft. 6 in. long, and 7 ft. 4 in. wide; and what will the stones cost at 25 cents a piece? *Ans.* 33 stones; cost \$8.25.

11. How many wine gallons will a vessel hold, that is 6 ft. 5 in. long, 4 ft. 6 in. wide, and 2 ft. 5 in. deep?

Ans. 522 gallons.

12. Find the value of two oak planks at $6\frac{1}{4}$ cents per square foot, each of which is 17 ft. 4 in. long, and the width of one is 1 ft. 5 in., and the width of the other is 10 inches?

Ans. \$2.$43\frac{3}{4}$.

13. How many feet of boards can be made from a stick of timber that is 18 feet long, 20 inches wide, and 16 inches thick, if we allow that $\frac{1}{4}$ of it is consumed in sawing?

Ans. 360 square feet.

14. At \$3 per cord, what is the value of a load of wood which is 12 feet long, 3 ft. 4 in. wide, and 2 ft. 3 in. high?

Ans. \$2.11, nearly.

15. How many bushels of grain will a bin contain that is 5 ft. 4 in. deep, 6 ft. 8 in. long, and 5 ft. 3 in. wide, their being 2150.4 cubic inches in a bushel? *Ans.* 150 bushels.

16. A plank is 13 ft. 6 in. long, and a carpenter wishes to slit off from it 12 square feet; at what distance from the edge must the line be struck? *Ans.* $10\frac{2}{3}$ inches.

17. At 15 cents per cubic yard, what will it cost to excavate

the earth for a cellar, which is to be 30 feet long, 24 feet wide, and 4 feet 8 inches deep? *Ans.* \$18.66$\frac{2}{3}$.

18. At 8 cents per square foot, what will it cost to purchase the stone for making a walk, 3 feet 6 inches wide, around a grass plat, which is 36 feet 8 inches long, and 18 feet 4 inches wide? *Ans.* \$34.72.

19. How many yards of paper that is $\frac{5}{8}$ of a yard wide, will be required for the walls of a room that is 20$\frac{5}{8}$ feet long, 11$\frac{1}{3}$ feet wide, and 12$\frac{1}{2}$ feet high, and what is its cost at 3 cents a yard? *Ans.* 140$\frac{1}{3}$ yards; cost \$421.

20. A cubic foot of wood weighs 11$\frac{1}{1}\frac{9}{1}$ pounds; what is the weight of a beam 24 feet long, 2$\frac{3}{4}$ feet wide, and 2$\frac{1}{2}$ feet thick, and what is its cost at 4 cents per cubic foot?

Ans. 1965 pounds; cost \$6.60.

21. What length of carpet that is $\frac{5}{8}$ of a yard wide will be required to cover the floor of a room that is 29$\frac{1}{2}$ feet long, and 11 feet and 3 inches wide; and what will the carpet cost at \$1.37$\frac{1}{2}$ per yard? *Ans.* 59 yards; cost \$81.12$\frac{1}{2}$.

22. It is found that 288 yards of paper, 2 feet and 8 inches wide, will cover the walls of a room: how many yards of paper that is 2 feet and 3 inches wide, will be required to cover the walls of this room; and what will the paper cost at 56$\frac{1}{4}$ cents per yard? *Ans.* 341$\frac{1}{3}$ yards; \$192.

23. What is the length of a room, whose breadth is 11 feet and 11 inches, if it takes 17 sq. yds. 2 ft. 131 in. to cover the floor? *Ans.* 13 ft. 1 in.

24. If a beam which is 10 inches wide, 8 inches deep, and 5 ft. 6 in. long, weigh 924 pounds, find the length of another beam, the end of which is a square foot, which shall weigh 2240 pounds. *Ans.* 7 ft. 4$\frac{8}{9}$ in.

CHAPTER XIV.

INVOLUTION AND EVOLUTION.

INVOLUTION.

108. The product obtained by using the same number as a factor two or more times, is called a power of that number. When a number is used as a factor *twice*, the product is called the *second* power; when it is used as a factor *three* times, the product is called the *third* power, and so on. The first power of a number, is the number itself.

The second power of a number is frequently called the *square* of that number, for the reason that the area of a square surface is found by multiplying the number which expresses the length of each of its four sides by itself. The third power of a number is often called the *cube* of that number, for the reason that the solidity of a cube is found by raising the number which expresses the length of each of its edges, to the *third* power.

Powers are denoted by means of small figures, which are placed to the right and above the number which is to be raised to a power. Thus the *third* power of 4 is expressed 4^3; the *fifth* power of 7 is expressed 7^5. These small figures are called *exponents* or *indices*.

Involution is the process of raising numbers to any required

power. From this definition, it follows that a number may be involved to any required power, by the following

RULE.

Employ the given number as a factor as many times as there are units in the exponent which denotes the required power, and the product of these equal factors, is the power sought.

EXAMPLES.

1. What is the second power of 25? *Ans.* 625.

2. What is the fourth power of $3\frac{1}{2}$* *Ans.* $12\frac{1}{4}$

3. What is the third power of 0.5? *Ans.* 0.125.

4. What is the sixth power of $1\frac{1}{2}$? *Ans.* $11\frac{25}{64}$.

5. What is the square of $3\frac{5}{8}$? *Ans.* $13\frac{9}{64}$.

6. What is the cube of 0.12? *Ans.* 0.001728.

7. What is the square of $10\frac{1}{5}$? *Ans.* 104.04.

8. Find the sum of the cube of 0.21, and the square of $3\frac{1}{4}$? *Ans.* 10.571761.

9. Find the sum of the square of $3\frac{7}{8}$, and third power of $\frac{1}{4}$. *Ans.* $15\frac{1}{32}$.

10. Find the difference of the square of $3\frac{1}{3}$, and the cube of $1\frac{2}{3}$. *Ans.* $6\frac{13}{27}$.

11. Find the product of the third power of $3\frac{1}{2}$, and the square of $2\frac{4}{7}$. *Ans.* $283\frac{1}{2}$.

---

* NOTE.—Reduce the mixed number to an improper fraction, and then apply the rule.

12. Divide the third power of $4\frac{1}{5}$ by the third power of $2\frac{1}{3}$.

Ans. $5\frac{104}{125}$.

EVOLUTION.

109. Every number may be regarded as being the product of a certain number of equal factors, and the process of finding one of these equal factors is called *Evolution.*

One of the equal factors of a number is called the *root* of that number. When a number is resolved into two equal factors, one of these factors is called the second or *square* root of that number; when it is resolved into three equal factors, one of these factors is called the third or *cube* root of that number; when it is resolved into four equal factors, one of the equal factors is called the fourth root; and so on.

Roots are denoted by means of *fractional* exponents, and the radical sign $\sqrt{\ }$ Thus, the square root of 5 is written $5^{\frac{1}{2}}$, or $\sqrt{5}$. The cube root of 7 is written $7^{\frac{1}{3}}$, or $\sqrt[3]{7}$. The index of the root is written over the radical sign. The sign $\sqrt{\ }$ when used alone denotes the square root.

When an indicated root of a number cannot be exactly expressed by figures, it is called a *surd.* Thus $\sqrt{5}$, $\sqrt[4]{7}$, and $\sqrt[5]{16}$ are *surds.* Their roots can only be obtained approximately, but the approximation may be carried to any extent, so that we can always obtain a number which shall differ from the true root of a number by less than any assignable quantity.

EXTRACTION OF THE SQUARE ROOT.

110. In order to deduce some convenient rule for the extraction of the square root of any number, we will first square some

number, and then seek to reverse the process by finding the square root of the square thus found. We will take the number 45, and observe that $45 = 40 + 5$. For squaring this number, we have the following

OPERATION.

$$\begin{array}{r}
40+\ 5 \\
\underline{40+\ 5} \\
200+25 = \text{the product of } 40+5 \text{ by } 5. \\
\underline{1600+200\qquad} = \text{the product of } 40+5 \text{ by } 40. \\
1600+400+25 = \text{the square of } 45 = 2025.
\end{array}$$

Here we may observe:

I. *That the square of any number consisting of two figures is equal to the square of the tens, increased by twice the product of the tens by the units, and the square of the units.*

II. *The square of* 45 *consists of twice its number of figures, and by squaring a series of numbers, we shall observe that the square of any number will contain twice as many figures, or one figure less than twice as many, as the number itself contains. Hence, if we commence at the unit figure of a number, and point it off into periods of two figures each, by placing a dot over every second figure, the number of periods thus formed will be equal to the number of figures in the root of that number.*

We will now endeavor to find the square root of 2025.

Since the square of tens produces *hundreds*, and the square of units produces only units, or tens and units, it follows that the square of the figure which expresses tens in the root cannot exceed the greatest square that can be taken from 20; and it is plain that the square of this figure in the root cannot be less than the greatest square that can be taken from 20. Hence,

the first figure in the root is 4, which is the square root of 16, the greatest square number that can be taken from 20.

We now subtract the square of 4 tens from 2025, and obtain for a remainder 425. From what has been shown, this remainder is equal to twice the product of the tens by the units increased by the square of the units. Since twice the product of the tens by the units consists of tens, the two left hand figures of the remainder express this product, either exactly or approximately. If the square of the unit figure in the root contains no tens, then 42 must exactly express twice the product of the tens by the units, but if the square of this unit figure does contain tens, then 42 must express a little more than the product of twice the tens by the units. This excess being comparatively small, we can determine, by way of trial, the unit figure of the root by dividing 42 by twice the tens, or 8. We find that 8 is contained 5 times in 42, with a remainder of 2. Hence we conclude that the next figure in the root must be 5, or nearly 5. If 5 is the unit figure, then the product of twice the tens plus 5 by 5 must equal the remainder 425. Twice the tens plus 5 may be found by annexing 5 to 8. Now, $85 \times 5 = 425$. Hence, the root is 45.

This process of extracting the square root of 2025, is exhibited in the following

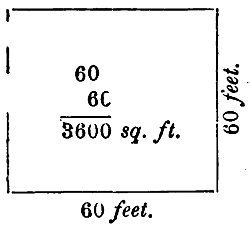
OPERATION.

$$
\begin{array}{rl}
\textbf{Number.} & \textbf{Root.} \\
\dot{2}0\dot{2}5 & (45 \\
\underline{16} & \\
85)425 & \\
\underline{425} & \\
0 &
\end{array}
$$

111. The process of extracting the square root of a number may be illustrated by a geometrical diagram. For this purpose, take the number 4225, and suppose it to represent 4225 square feet. We are required to find the side of a square that contains 4225 square feet.

By what was shown in the last article, the number of feet in a side of this square, is expressed by two figures, and the first figure of this root is 6, which expresses *tens*. We now construct a square, the side of which is 60 feet. This square is represented by Fig. 1.

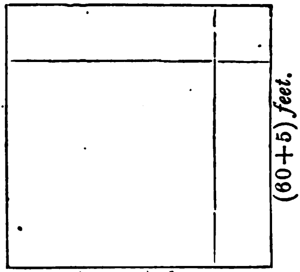
Fig. 1.



By subtracting 3600 from 4225 we have for a remainder 625 square feet. The square represented by figure 1 must now be enlarged by 625 square feet. The square represented by figure 1 may be enlarged, so that the resulting figure may be a square, by means of two equal rectangles of the length of the square, and a small square, the side of which is equal to the width of one of the rectangles. The square thus enlarged is represented by Fig. 2.

We must now determine the width of these rectangles.

Fig. 2.



These rectangles, when arranged, as represented in figure 3, form a rectangle 120 feet long. And it is evident that this rectangle contains much the larger part of the 625 square

Fig. 3.

feet. The width, therefore, of this rectangle will not differ much from the width of one of the same length, that contains 625 square feet; and since the width of any rectangle is found by dividing its area by its length, we can determine, by way of trial, the width of the rectangle represented by Fig. 3, by dividing 625 by 120, or by dividing 62 by 12. We find that 12 is contained 5 times in 62, with a remainder of 2. Hence, we conclude, that the width of the rectangle is five feet, or nearly 5 feet. If 5 feet is the width of the rectangle, then we can form from the two equal rectangles and the square, a rectangle represented by figure 4, which is 125 feet

Fig. 4.

long, 5 feet wide, the area of which is $125 \times 5 = 625$ square feet. This number of square feet being the number that remained after subtracting the area of the square represented by Figure 1, from 4225, it follows that the side of a square which contains 4225 square feet, is 65 feet. Whence 65 is the square root of 4225.

The above process of extracting the square root of 4225 is exhibited in the following

OPERATION.

```
Number.  Root.
  4225  (65
  36
125)625
    625
      0
```

Here we first point the number off into periods of two figures each. Having done this, we proceed as follows:

The greatest square that can be taken from 42, the left hand period, is 36. The square root of 36 is 6, which we place in the root as being its first, or left hand figure.

We now subtract 36 from 42, and to the difference we annex the next period, and thus obtain 625, which we shall call the first *dividend.*

Having found the first dividend, we then double the figure in the root, and obtain 12, which we shall call the *trial di-*

visor. By rejecting the right hand figure of the dividend, we find that 12 is contained in the remaining part of the dividend 5 times. We place this quotient in the root for its next figure, and also to the right of the trial divisor, and thus form the true divisor, 125. We now multiply the true divisor by the last figure in the root, and subtract the product from the dividend, and as there is no remainder, the operation is finished.

Sometimes one of the figures of the root is a cipher. As an example of this kind, let it be required to extract the square root of 93025. For the extraction of the square root of this number, we have the following

OPERATION.

```
      Number.  Root.
        . . .
       93025(305
       9
       —
605)3025
       3025
       ————
          0
```

We point the number off into periods, as in the last example. The greatest square contained in 9 is itself, the root of which we place at the right for the first figure in the required root.

From the first period we subtract the square of the figure in the root; the remainder is nothing. We bring down 30 for the first dividend, and double the figure in the root for the first trial divisor. We find that 6, the trial divisor, is not contained in 3, the dividend with its right hand figure rejected, and we place a cipher at the right of 3, and also at the right of 6.

To 30 we annex 25, and the dividend is 3025. We take 60

as a trial divisor, and find that it is contained in 302 5 times. Annexing 5 to the figures in the root, and to the trial divisor, and multiplying the complete divisor 605, by 5, and subtracting the product from the dividend, 3025, the operation is finished.

From the preceding solutions and explanations, we deduce the following rule for the extraction of the square root.

RULE.

I. *Separate the given number into periods of two figures each, by setting a point over the unit figure, another over the figure expressing hundreds, and so on over every second figure.*

II. *Find the greatest square in the first or left hand period, and set its square root at the right of the given number, as the first or left hand figure of the required root.*

III. *Subtract the square thus found from the first period, and to the remainder annex the figures in the following period for a dividend.*

IV. *Double the figure in the root for a trial divisor, and seek how many times it is contained in the dividend, omitting its right hand figure. Annex the quotient to the right of the figure in the root, and also to the right of the trial divisor for a true divisor. Multiply the true divisor by the last figure in the root, subtract the product from the dividend, and to the remainder bring down the next period for a second dividend.*

V. *Double the part of the root already found for a second trial divisor, with which proceed as before; and so on till the required root is found.*

112. Numbers which are not square numbers, are called im-

perfect squares. In finding the roots of such numbers, periods of ciphers may be annexed, as forming so many periods of decimals, but we must be careful to observe that the number of decimal places in the root must equal the number of decimal periods employed. This follows from the definition of square root, and from the rule for the multiplication of decimals. In this way we can find the root of an imperfect square to any required degree of exactness.

113. To find the square root of a common fraction, divide the square root of its numerator by the square root of its denominator, or express the division in the form of a fraction. Or we may find the equivalent decimal fraction, and then take the root of this decimal for the required root. It is advisable to take this course when the terms of the fraction are surd numbers.

Sometimes the terms of a fraction are surds, when an equivalent fraction can be found whose terms are square numbers. Thus, the terms of the fraction $\frac{12}{27}$ are surds, but the terms of its equivalent fraction, $\frac{4}{9}$, are square numbers. The square root of $\frac{12}{27}$ equals the square root of $\frac{4}{9}$, which is $\frac{2}{3}$. To determine whether or not the square root of a fraction can be exactly expressed by another fraction, reduce the given fraction to its lowest terms, and if these terms are squares, the square root of the given fraction can be so expressed.

114. In extracting the square root of a mixed number which is composed of a whole number and a decimal fraction, we must commence at the *unit* figure in separating it into periods, and set a point over every alternate figure, both to the right and

the left. If the number of decimal places is not even, a cipher can be placed at the right of the decimal.

115. In the application of the rule, it will sometimes happen that the product of a true divisor and a figure in the root will exceed the dividend which corresponds to these numbers. In such a case, the figure in the root is too large, and it must be diminished. In the following operation for the extraction of the square root of 729, the second figure in the root, obtained by means of the trial divisor, is too large.

OPERATION.

Number. Root.

$$\begin{array}{r|l} \dot{7}2\dot{9} & (27 \\ 4 & \\ \hline 47)329 & \\ 329 & \\ \hline 0 & \end{array}$$

Here the trial divisor, 4, is contained 8 times in 32. Now, $48 \times 8 = 384$, and as this product is larger than the dividend, we must diminish the root figure 8, as in the operation.

116. By means of the square root, we can obtain the 4th root, the 8th, the 16th, or the root of any power* whose exponent is some power of 2.

For example, let it be required to extract the fourth root of the number 390625. To do this, we must find a number of which 390625 is the fourth power. Now it is plain that the

* NOTE.—Any number may be regarded as being a power of some number. Thus, 47 may be regarded as being the fourth power of a number which we can find by extracting the fourth root of 47.

fourth power of any number is equal to the square of the second power of that number. Hence, the square root of 390625 must be the square of the required root, and the square root of the square root of 390620 is the root sought. We have, then, to find the square root of 390625, the following

OPERATION.

To find the square root of 390625.	To find the root of 625.
390625(625	625(25, the root required.
36	4
122)306	45)225
244	225
1245)6225	0
6225	
0	

117. The square root of the product of two or more factors is equal to the product of their square roots. Thus, the square root of 100, which is the product of the two factors 4 and 25, is equal to $\sqrt{4}$ multiplied by $\sqrt{25}$. For, $\sqrt{4}\times\sqrt{25}=2\times5=10$, and $\sqrt{100}=10$.

On this principle, we can sometimes find the square root of a number without the labor of applying the preceding rule. If the number can be resolved into factors, each of which is a *square number*, we can take the square root of each factor, and the product of these roots will be the root required. Or, if there is an *even* number of each of the different prime factors of the given number, we can find its root by taking one half of the number of factors in each set of different factors, and finding the product of the selected factors for the required root.

For example, to find the square root of $5\frac{1}{6}$, we observe that $576=9\times 64$; hence, $\sqrt{576}=\sqrt{9}\times\sqrt{64}=3\times 8=24$. Or, since $576=2\times 2\times 2\times 2\times 2\times 2\times 3\times 3$, $\sqrt{576}=\sqrt{2\times 2\times 2\times 2\times 2\times 2}\times\sqrt{3\times 3}=2\times 2\times 2\times 3=24$.

EXAMPLES.

1. Extract the square root of 186624. *Ans.* 432.
2. Extract the square root of 77841. *Ans.* 279.
3. Extract the square root of 10291264. *Ans.* 3208.
4. Extract the square root of 195364. *Ans.* 442.
5. Extract the square root of 328329. *Ans.* 573.
6. Extract the square root of 0.0676. *Ans.* 0.26.
7. Extract the square root of 87.65. *Ans.* 9.3622.
8. Extract the square root of 861. *Ans.* 29.34.
9. Extract the square root of 984064. *Ans.* 992.
10. Extract the square root of 5. *Ans.* 2.236.
11. Extract the square root of 0.5. *Ans.* 0.7071.
12. Extract the square root of 0.331776. *Ans.* 0.576.
13. Extract the square root of $2\frac{1}{4}$*. *Ans.* $1\frac{1}{2}$.
14. Extract the square root of $17\frac{3}{8}$†. *Ans.* 4.1683.
15. Extract the square root of $51\frac{21}{25}$. *Ans.* $7\frac{1}{5}$.
16. Extract the square root of $6\frac{2}{5}$. *Ans.* 2.5298.

---

* NOTE—Observe that $2\frac{1}{4}=\frac{9}{4}$; hence, $\sqrt{2\frac{1}{4}}=\sqrt{\frac{\ }{4}}=\frac{3}{2}$.

† NOTE.—Here, by reducing $\frac{3}{8}$ to a decimal fraction, we find $17\frac{3}{8}=17.375$; hence, $\sqrt{17\frac{3}{8}}=\sqrt{17.375}$. See Art. 114.

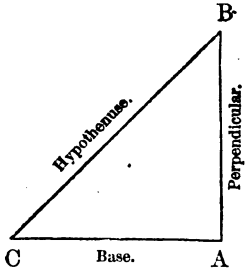
17. Extract the square root of $1\frac{3}{80}$. *Ans.* 1.01858.
18. Extract the square root of $13\frac{1}{5}$. *Ans.* 3 6332.
19. Extract the square root of $27\frac{9}{16}$ *Ans.* $5\frac{1}{4}$.
20. Extract the square root of $\frac{3}{13}$.

$$Ans. \sqrt{\frac{39}{169}}=\frac{\sqrt{39}}{13}=\frac{6.244}{13}.$$

APPLICATION OF THE SQUARE ROOT.

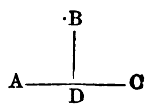
118. Application of the square root is frequently made in the solution of questions which involve the principles of geometry. We shall here exhibit such an application in finding a side of a right-angled triangle when two of its sides are given.

A right-angled triangle is a rectilineal figure having their sides and their angles, one of which is a right angle.* The fig-



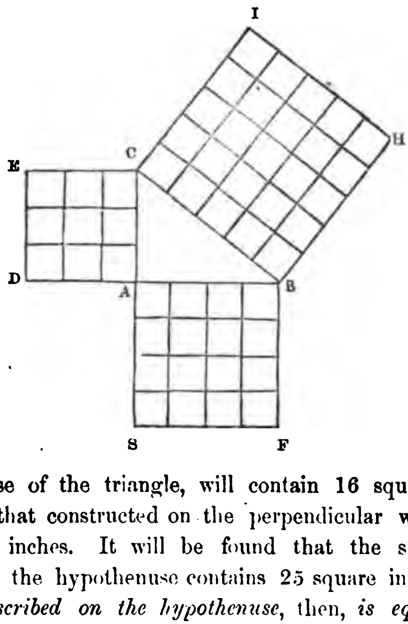
ure A B C is a right-angled triangle in which the angle B A C

* NOTE.—When one straight line meets another straight line so as to make the adjacent angles equal to each other, the angles are called right angles. Thus, if the angles A D B and D B C are equal, each is a right angle. By the term angle is meant the difference of direction of two lines.



is the right angle. The side B C, opposite to the right angle, is called the hypothenuse; the side A C is called the base; the side A B is called the perpendicular.

Suppose that we have a right-angled triangle, the base of which is 4 inches, and the perpendicular 3 inches. If we construct squares upon the three sides of the triangle, as represented in the following figure, it is plain that the square constructed



on the base of the triangle, will contain 16 square inches, and that that constructed on the perpendicular will contain 9 square inches. It will be found that the square constructed on the hypothenuse contains 25 square inches. *The square described on the hypothenuse*, then, *is equal to the sum of the squares described on the base and perpendicular*, and in any other right-angled triangle, the same relation between the square described on the hypothenuse, and the

sum of the squares described on the other two sides, will exist.* Hence,

The hypothenuse may be found by extracting the square root of the sum of the squares described on the base and perpendicular; and

Either side adjacent to the right angle may be found by extracting the square root of the difference of the squares described on the hypothenuse and the other side adjacent to the right angle.

For example, if the base and perpendicular are 4 and 3, respectively, as represented in the figure, we proceed as follows to find the hypothenuse:

The square of the base is 4^2	$=16$
The square of the perpendicular is 3^2	$=\ 9$
The sum of the squares of these two sides	$=25$
Hence, the hypothenuse	$=\sqrt{25}=5.$

EXAMPLES.

1. What is the distance around a square field that contains 40 acres; and what is the length of the diagonal of such a field? *Ans.* Distance 320 rods; diagonal 113.137 rods.

2. A surveyor is requested to lay out a piece of ground containing 80 acres in the form of a rectangle, the length of which shall be twice its width; what are its length and width?

Ans. 160, and 80 rods.

---

* NOTE.—For a demonstration of the proposition that the square described on the hypothenuse of a right-angled triangle is equal to the sum of the squares described on the other two sides, the student is referred to a treatise on geometry. This celebrated and important proposition of elementary geometry is susceptible of at least thirty different demonstrations.

3. How many more rods of fence will be required to fence a rectangular lot which contains 25 acres, and whose length is twice its width, than is required to fence a square lot containing the same number of acres? *Ans.* 15.346 rods.

4. Find the solidity of a cube the diagonal of which is 18 inches.* *Ans.* 1122.36 cubic inches, nearly.

5. In the preceding question, find the diagonal of one of the faces of the cube. *Ans.* 14.697 inches, nearly.

6. In example 4, find the area of the six faces of the cube. *Ans.* 648 square inches, nearly.

7. The diagonal of a cube is 675 inches, and it is required to find the diagonal of a rectangular solid, the length of which is twice that of one of the edges of the cube, and the width and height of which are each equal to one half of its length. What is the length of that diagonal?

Ans. 954.614 inches, nearly.

8. In the preceding example, find the area of the six faces of the solid. *Ans.*

9. The length of a ladder is 65 feet, and when the foot of the ladder is placed 52 feet from a house, and the other end against the wall of a house, it is found that the top just reaches

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* Note.—The following question is proposed to those who have some knowledge of adfected quadratic Equations:

The diagonal of a cube exceeds one of its edges by 4 inches; it is required to find the solidity.

It may be remarked, that, by adopting a proper notation, a solution of this question may be given which does not involve an adfected quadratic equation.

to the bottom of a window in the fourth story of the house. What is the distance from the ground to this window?

Ans. 39 feet.

10. Two sides of a triangle are 20 and 25 feet long, respectively; what must be the length of the third side, in order that the angle included by the two given sides may be a right angle? *Ans.* 32 feet, nearly.

11. A ladder, 40 feet long, was so placed in a street as to reach a window 33 feet from the ground, and when it was turned to the other side, without changing the position of its foot, it reached a window 21 feet high? Find the width of the street? *Ans.* 56.6 feet.

12. A castle which is 45 feet high, is surrounded by a ditch, which is 24 feet wide; what must be the length of a ladder that will reach from the outside of the ditch to the top of the castle? *Ans.* 38 feet, nearly.

13. Two ships start from the same point, and one sails directly south at the rate of 8 miles per hour, and the other sails directly east, at the rate of 10 miles per hour; what distance are the ships from each other at the end of 6 hours?

Ans. 76.8 miles.

14. The length, width, and thickness of a piece of marble are to each other as 3, 2, and 1, respectively; and it cost as many cents per cubic foot as there are feet in its thickness, and the whole cost was $15.36. What were its dimensions?

Ans. Length 12, width 8, and thickness 4 feet.

15. How many more rods of fence will be required to enclose a lot containing 30 acres, the length of which is three times its width, than are required to enclose a square lot containing the same amount of land? *Ans.* 42.87 rods.

16. A park contains 10 acres, and in its centre there is a reservoir in the form of a square, the area of which is equal to $\frac{1}{10}$ of the area of the whole park. What is the length of one side of the reservoir, and what is the length of a string reaching from one corner of the park to the opposite corner of the park? *Ans.* 12.649, and 40 rods.

17. In the preceding example, find the length of the diagonal of the reservoir, and also that of the diagonal of the park.

Ans.

18. What will be the expense of enclosing 15 acres in the form of a square, at the rate of 1.375 a rod? *Ans.* \$269.42

EXTRACTION OF THE CUBE ROOT.

119. To extract the cube root of a number is to find one of the three equal factors into which that number may be resolved. Thus the cube root of 27 is 3, since $3\times3\times3=27$.

In order to discover a convenient rule for the extraction of the cube root, let us cube a number, and then seek to reverse the process by extracting the cube root of the cube of that number.

If we cube 84, which is equal to 8 tens plus 4 units, or 80 + 3, we shall have the following

OPERATION.

$$\begin{array}{l}
80+4 \\
\underline{80+4} \\
80^2+\quad 80\times4 \\
\qquad\quad \underline{80\times4+4^2} \\
80^2+2\times80\times4+4^2=\text{the square of } 80+3\text{, or } 83. \\
\underline{80+3} \\
80^3+2\times80^2\times4+\quad 80\times4^2 \\
\qquad\quad \underline{80^2\times4+2\times80\times4} \\
80^3+3\times80^2\times4\times3\times80\times4^2+4^3=592704\text{, cube of } 80+4.
\end{array}$$

That is,

The cube of any number consisting of tens and units, is equal to the cube of the tens, plus three times the square of the tens, multiplied by the units, plus three times the tens multiplied by the square of units, plus the cube of the units.*

Let us now seek to extract the cube root of $80^3+3\times80^2\times4+3\times80\times4^2+4^3=592704$.

We can find the first term in this root by extracting the cube root of 80^3. The cube root of 80^3, is 80, or 8 tens. Now, if we subtract the cube of 80 from $80^3+3\times80^2\times4+3\times80\times4^2+4^3$, the remainder is $3\times80^2\times4+3\times80\times4^2+4^3=80704$. If we divide the first term of this remainder by 3×80^2, or three times the square of the tens, the quotient will be 4, which is the unit figure in the required root.

It is plain that $3\times80^2\times4$, or three times the square of the tens multiplied by the units, *constitutes by far the larger part of the remainder* 80704. *If, therefore, we divide* 80704 *by* $3\times80^2=19200$, *the first figure in the quotient must be the second figure in the root, or a number a very little larger than this part of the root.* We may, therefore, regard 19200 as a *trial divisor* for determining the next figure in the root. We find that the first figure in the quotient of 80704 divided by 19200 is 4, and this is the unit figure in the root.

Let us now endeavor to determine in what way we can find a *complete divisor* of 80704, that is, a divisor that is contained in it exactly 4 times.

If we divide each term of the remainder, $3\times80^2\times4+3\times80$

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* NOTE.—The word *plus* denotes addition. It is used here for the words *added to.*

$\times 4^2+4^3$, by 4, we shall find that the quotient is $3\times 80^2+3\times 80\times 4+4^2=20176$; hence, 20176 is the *complete* or *true divisor* of 80704.

This process of returning to the cube root of 592704, is exhibited in the following

OPERATION.

Col. I.	Col. II.	Number.	Root.
		592704	(80×4=84
80	6400	512000	
160	19200 *trial divisor.*	80704	=1*st dividend.*
244	20176 *true divisor.*	80704	
		0	

To find the cube of 80, we place 80 for the first term of a column designated Col. I., and its square for the first term of a column designated Col. II. Then we multiply the term in Col. II. by 80, and obtain the cube of 80, which we subtract from 592704, and obtain for the remainder 80704, which is the first dividend.

To find the trial divisor, we add 80 to the first term of Col. I., and obtain for the sum 160, which is the second term in Col. I. We then multiply the second term of Col. I. by 80, and add the product, 12800, to the first term in Col. II., and the sum* thus obtained is the trial divisor. The trial divisor is contained 4 times in the dividend, and the 4 we add to 80, it being a part of the root.

To find the true divisor, we add 80 to the second term in

* NOTE.—The additions and multiplications that are to be made in forming the successive terms in the two columns, can generally be carried on *mentally*.

Col. I., and obtain for the sum 240, and to this sum we add 4, and obtain for the third term in Col. I., 244. We then multiply 244 by 4, and add the product to the trial divisor; the sum is the true divisor, and we then multiply the true divisor by 4, and subtract the product from the dividend. This last step finishes the operation.

By omitting the ciphers at the right of the terms in the two columns, the operation may be represented as follows:

Col. I.	Col. II.	Number.	Root.
		592704	(84
8	64	512	
16	192	80704	
244	20176	80704	
		0	

The cube of any number *contains three times as many figures, or two figures less* than three times as many as are contained in the number itself. Hence, if we commence at the unit figure of any number, and point it off into periods of three figures each, by setting a point over the unit figure, and one over every third figure to the left, the number of periods in the given number will be equal to the number of figures in its root.

When the root of a number contains more than two figures, we can form the successive true and trial divisors in the same manner that the first trial and true divisors were formed. Hence, for the extraction of the cube root of any whole number, we have the following

RULE.

I. *Divide the number into periods of three figures each by*

setting a point over the unit figure, and one over every third figure to the left.

II. *Find the greatest perfect cube that can be taken from the first or left hand period, and place its root on the right of the number for the first or left hand figure in the required root. Then place this root figure for the first term of a column designated Col. I., and its square for the first term of a column designated Col. II. Multiply the term in Col. II. by the figure in the root, and subtract the product from the first or left hand period in the given number. To the remainder annex the next period for the first dividend.*

III. *Add the first figure in the root to the first term in Col. I. and the sum is the second term in this column. Multiply the second term in Col. I. by the figure in the root, and add the product to the first term in Col. II. The sum is the second term in this column, or the first trial divisor.*

IV. *See how many times the trial divisor, with two ciphers annexed, is contained in the first dividend, and take the quotient for the next figure in the root. Add the first figure in the root to the second term in Col. I., and to the sum annex the second figure in the root for the third term in this column. Multiply the third term in Col. I. by the second figure in the root, and add the product, two places advanced to the right, to the first trial divisor, and the sum is the first true divisor. Multiply the true divisor by the second figure in the root, and subtract the product from the first dividend, and to the remainder annex the next period for the second dividend.*

V. *To find the second trial divisor add the second figure in the root to the third term in Col. I., and the sum will be the fourth term in this column. Multiply the fourth term in this column by the second figure in the root, and add the pro-*

duct to the first true divisor; the sum will be the second trial divisor. With the second trial divisor, proceed as with the first, and so on till the required root is obtained.

In the application of this rule, it will sometimes happen that the product of the true divisor and the corresponding figure in the root, will exceed the dividend. In such a case, diminish the figure in the root, and then form a new true divisor. Thus, in the following operation for the extraction of the cube root of 50653, the trial divisor, 27, with two ciphers annexed, is contained 8 times in the dividend, but if we proceed to form the true divisor, we shall discover that it is not contained 8 times in the dividend; hence we diminish 8 by 1, and with 7 as the second figure in the root, we form another true divisor. We shall seldom meet with this difficulty after we have found the first two figures of the root.

Col. I.	Col. II.	Number.	Root.
		$\dot{5}06\dot{5}3$	(37
3	9	27	
6	27	23653	
97	3379	23653	
		0	

120. In extracting the cube root of a mixed number, which consists of a whole number and a fraction, we must commence, in pointing it off into periods, at the *unit* figure, and place a point over every third figure both to the right and the left hand. The root will contain as many decimal figures as there are decimal periods; for, by the rule for the multiplication of decimal fractions, the cube of any decimal fraction will contain *three* times as many decimal figures as the decimal fraction. If the number of decimal figures is not some multiple of 3, we can annex ciphers as decimal figures to supply the deficiency.

In relation to the extraction of the cube root of a fraction, and the cube root of a mixed number, we may make observations similar to those made for the extraction of the square root of a fraction and a mixed number.

121. We will now explain the rule for the extraction of the cube root of a number by means of geometrical diagrams.

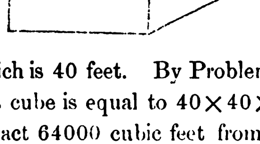
Let it be required to find the side of a cube which contains 91125 cubic feet.

Since the solidity of a cube is found by cubing the number which expresses the length of one of its edges, the solution of this question consists in finding the cube root of 91125.

By separating 91125 into periods, we find that there will be two figures in the root. The greatest perfect cube that we can subtract from 91, the left hand period, is 64, and its cube root is 4. Since there are two figures in the required root, the first figure, 4, must denote 4 tens, or 40.

We now form a cube, represented by Figure 1, the length

Fig. 1.



of the edge of which is 40 feet. By Problem II., Article 106, the solidity of this cube is equal to $40\times40\times40=64000$ cubic feet. If we subtract 64000 cubic feet from 91125 cubic feet, we shall obtain a remainder of 27125 cubic feet. We can,

then, enlarge the cube represented by Figure 1, by this number of cubic feet.

We can first enlarge the cube by placing on each of its three adjacent faces, one of three equal rectangular solids, such that the face of each of which will exactly cover the face of the cube. These three equal solids are represented by Figures 2, 3, and 4, and the cube thus enlarged, is represented by Fig. 5.

Fig. 2.

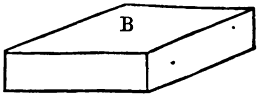


Fig. 3.

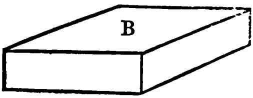


Fig. 4.

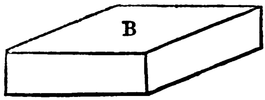
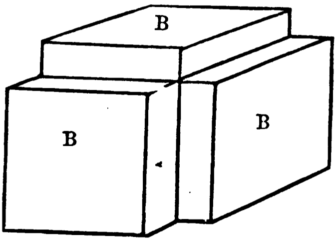


Fig. 5.



We can enlarge the solid represented by Fig. 5, by placing one of three equal rectangular solids in each of the vacant corners. The length of each of these three equal solids is equal to the length of an edge of the cube, and the width and thickness of

each are respectively equal to the thickness of the solids represented by Fig. 2, 3, and 4. These three solids are represented by Fig. 6, 7, and 8, and the enlarged solid is represented by Fig. 9.

Fig. 6.

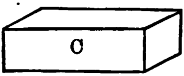


Fig. 7.

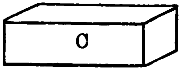


Fig. 8.

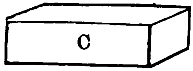
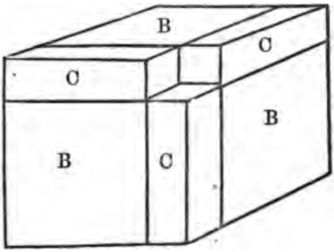
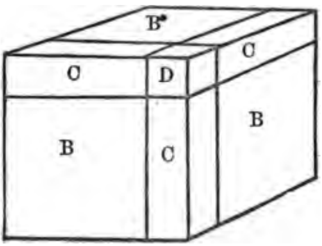


Fig. 9.



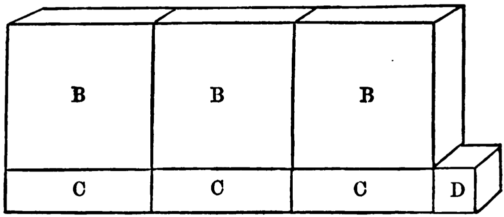
Now to render the solid represented by Figure 9, a cube, we have only to supply the vacant corner represented in the figure, by a small cube, the length of the edge of which is just equal to the thickness of one of the solids marked B. The solid thus enlarged becomes a *cube*, and is represented by Figure 10.

Fig. 10.



We have seen, then, that the cube marked A, can be enlarged so that the resulting figure may be a cube, by means of the seven solids represented by figures 2, 3, 4, 6, 7, and 8. These seven solids can readily be arranged so as to form one solid represented by Figure 11.

Fig. 11.



Since the length of each edge of the cube represented by Figure 1, will be increased by the thickness of the solid last formed, we can determine the size of the required cube by ascertaining what the thickness of the solid represented by Figure 11 must be, in order that it may contain 27125 cubic feet.

It is plain that if the last solid was 1 foot thick, it would contain as many *cubic* feet as there are *square* feet in its front surface; if it was 2 feet thick, it would contain twice as many cubic feet as there are square feet in its surface, and so on; that is, its solidity is equal to the product of the number of square feet in its surface, and its thickness. Hence, if we knew the number of square feet in its front surface, we could find its thickness by dividing its solidity, 27125 cubic feet, by this number of square feet.

Now it is clear that the number of square feet in the three front surfaces of the solids marked B, in Fig 11, constitute much the *larger* part of the front surface of this figure. Hence, we

can employ the number of square feet in these three surfaces as a *trial* divisor of 27125, and the first figure of the quotient of this division will express the thickness of the solid represented by Fig. 11, or it may express a little more than this thickness, since the divisor is a little smaller than the true divisor.

To find the trial divisor we first obtain the number of cubic feet in the first cube, and use two columns in the operation, as in the last article. We then observe that 1600, the first term

Col. I.	Col. II.	Number.	Root.
		91125(40	
40	1600	64000	5
80	4800 Trial divisor,	27125	
125	5425 True divisor.	27125	
		0	

of Col. II., is the number of square feet in the front surface of one of the solids marked B. Hence, if to this term in Col. II. we add the number of square feet in the front surface of two of the solids marked B, we shall obtain the trial divisor. Now two of these equal surfaces form a rectangle that is $40+40=$ 80 feet long, and 40 feet wide. Observe that the length of this rectangle may be found by adding 40, the part of the root already found, to the first term in Col. I. Its area$=80\times 40=$ 3200, and $1600+3200=4800$, the trial divisor.

The trial divisor is contained 5 times in 27125, with a remainder. We therefore suppose that the thickness of the solid represented by Fig. 11, is 5 feet.* Under this supposition, let us find its solidity.

* NOTE.—The reader must recollect that the *trial divisor*, as the term indicates, is only used for *approximating* to the thickness of the solid represented by Fig. 11.

To find the solidity of this solid, we must first find the area of its front surface. If to the trial divisor we add the sum of the areas of the front surfaces of the three solids marked C, and the front surface of the cube marked D, we shall obtain the surface required. By inspecting the figure, it is evident that these four surfaces form a rectangle which is $40+40+40+5=125$ feet long, and 5 feet wide. Now the length of this rectangle can be found by adding 40, the first part of the root, to the second term in Col. I., and then adding 5, the second figure in the root, to the sum. The area of the rectangle is (125×5) sq. feet$=625$ square. Hence, the required surface, or the true divisor, is 625 sq. feet$+4800$ sq. feet$=5425$ sq. feet. Whence, the solidity of the solid is (5425×5) cu. feet, or 27125 cubic feet. Hence it appears that the side of the required cube is $(40+5)$ feet$=45$ feet.

If we omit the ciphers at the right of the numbers in the two columns, and also the one in the root, in this process for extracting the cubic root of 91125, we may deduce from it the rule stated in the last article.*

122. The cube root of the product of two or more factors is equal to the product of their cube roots. For example, the cube root of $9824=8\times27\times64$, is equal to $\sqrt[3]{8}\times\sqrt[3]{27}\times\sqrt[3]{64}=2\times3\times4=24$. On this principle, we can find, without the application of the rule, the cube root of any number which can be resolved into small factors, each of which is a perfect cube; and any number can be so resolved, providing that is a perfect cube. These cubic factors can readily be discovered by resolving the given number into its prime factors.

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* NOTE.—For an algebraical explanation of the rule for extracting the cube root, see Elements of Algebra.

When a number contains a cubic factor, we can find its root, by multiplying the cube root of this factor by the cube root of the other factor of the number. For example, $\sqrt[3]{320}=\sqrt[3]{64}\times\sqrt[3]{5}=4\times\sqrt[3]{5}=4\times1.709976=6.839804$.

EXAMPLES.

1. What is the cube root of 48228544?

OPERATION.

Col. I.	Col. II.	Number.	Root.
		48228544	(364
3	9	27	
6	27	21228	
96	3276	19656	
102	3888	1572544	
1084	3931.36	1572544	
		0	

2. What is the cube root of 64481.201?

OPERATION.

Col. I.	Col. II.	Number.	Root.
		64481.201	(40.1
4	16	64	
8	48	481201	
1201	481201	481201	
		0	

3. What is the cube root of 123505992? *Ans.* 498.

4. What is the cube root of 190109375? *Ans.* 575.

5. What is the cube root of 458314011? *Ans.* 771.

6. What is the cube root of 483.736625? *Ans.* 7.85.

7. What is the cube root of 0.636056? *Ans.* 0.86.

8. What is the cube root of 0.979146657 *Ans.* 0.993.

9. What is the cube root of $\frac{8}{125}$? *Ans.* $\frac{2}{5}$.

10. What is the cube root of $\frac{3}{7}$? *Ans.* 0.753+.

11. What is the cube root of $28\frac{3}{4}$? *Ans.* 3.0635.

12. What is the cube root of $7\frac{1}{5}$? *Ans.* 1.93.

13. What is the cube root of $9\frac{1}{6}$? *Ans.* 2.0928.

14. What is the cube root of $71\frac{3}{4}$? *Ans.* 4.155.

15. What is the cube root of $32\frac{8}{\text{[illegible]}}$? *Ans.* 3.1987.

16. What is the cube root of 382657176? *Ans.* 726.

17. What is the cube root of 592704? *Ans.* 84.

18. What is the cube root of 15625? *Ans.* 25.

19. What must be the side of a cube, the solidity of which shall be equal to that of a rectangular solid 288 feet long, 216 feet wide, and 48 feet thick? *Ans.* 144 feet.

20. The length, breadth and thickness of a rectangular solid containing 32768 cubic feet, are to each other as the numbers 16, 4, and 1. What are the dimensions of this solid?

Ans. 128 feet long; 32 ft. wide; 8 ft. thick.

21. What is the side of a cube, the capacity of which is equal to that of a chest 2 feet 8 inches long, 2 feet 3 inches wide, and 1 foot 4 inches thick? *Ans.* 24 inches.

22. What must be the side of a cubical vessel that shall contain 216 wine gallons? *Ans.* 4.0679 feet.

23. How many bricks, each of which is 8 inches long, 4

inches wide and 2 inches thick, will be required to construct the walls for a cubical cistern that will contain 1000 wine gallons, if we allow that the cistern is open at the top, that $\frac{1}{5}$ of the wall is made of plaster, and that it is 1 foot thick?

Ans. 3793.

24. In the preceding example, what will the bricks cost at $4.75 per thousand? *Ans.*

25. A rectangular solid contains 3456 cubic inches, and its breadth and height are each equal to one-half of its length. It is required to find the length of the diagonal of this solid.

Ans. 29.39 inches.

26. A rectangular vessel contains 1265625 cubic inches, and its width and height are each a third part of its length. How many more square inches of zinc will be required to line this solid, than it will require to line one of the same capacity in the form of a cube? *Ans.* 8547.13.

It may be shown by the aid of the Differential Calculus, that the surface of a rectangular solid of a given volume, will be *minimum* or *the least possible* when that solid is a cube.

CHAPTER XV.

PROGRESSIONS.

ARITHMETICAL PROGRESSION.

123. An arithmetical progression is any series of numbers which increase or decrease by the addition or subtraction of the same number. Thus, the series of numbers 1, 4, 7, 10, 13, &c., each of which, except the first, is found by adding 3 to the preceding term in the series, is an *increasing* arithmetical progression; and the numbers 13, 11, 9, 7, 5, 3, 1, each of which, after the first, is obtained by subtracting 2 from the preceding number, form a *decreasing* arithmetical progression.

124. In an arithmetical progression, there are *five* things to be considered, namely, the *first term*, the *common difference*, the *number of terms*, the *last term*, and the *sum of all the terms*. Any three of these being given, the other two may be found.

PROBLEM I.

In an arithmetical progression, having given the first term, the common difference, and the number of terms, it is required to find the last term.

If we examine any increasing arithmetical progression, we shall find that the second term is equal to the first term, increased by the common difference, that the third term is equal to the first term increased by *twice* the common difference; and that, in general, any term is equal to the first term increased by the product of the common difference, and the number of terms less one. Hence, to find the last term, *add the first term to the product of the common difference and the number of terms less one.*

PROBLEM II.

Having given the first term, the last term, and the number of terms, it is required to find the sum of all the terms.

Let us take the arithmetical progression 1, 5, 9, 13, 17, and add it to itself after reversing the order of the terms. For making this addition, we have the following

OPERATION.

$$\begin{array}{ccccccc} 1 & + & 5 & + & 9 & + & 13 \\ \underline{17} & + & \underline{13} & + & \underline{9} & + & \underline{5} \\ 18 & + & 18 & + & 18 & + & 18 \end{array}$$

Hence, twice the sum of the series is equal to 18×4; therefore, the sum itself is $\frac{1}{2} \times 18 \times 4 = 9 \times 4$. But 9 is one-half the sum of the extremes, and 4 is the number of terms; hence, *the sum of the terms is equal to one-half the sum of the extremes multiplied by the number of terms.*

EXAMPLES.

1. One extreme is 3, the other 15, and the number of times is 7. What is the sum of the series? *Ans.* 63.

2. One extreme is 5, the other 93, and the number of terms is 49. What is the sum of the terms? *Ans.* 2401.

3. One extreme is 147, the other $\frac{3}{4}$, and the number of terms is 97. What is the sum of the series?

Ans. 7165.875.

4. The extremes of an arithmetical series are 21 and 497, and the number of terms is 41. What is the common difference? * *Ans.* 11.9.

5. The extremes of an arithmetical progression are $127\frac{25}{28}$ and $9\frac{1}{7}$, and the number of terms is 26. What is the common difference? *Ans.* $4\frac{3}{4}$.

6. In an arithmetical series, the extremes are 96 and 12, and the common difference is 6. What is the number of terms?

Ans. 15.

7. In an arithmetical series, the extremes are 14 and 32, and the common difference is 3. What is the sum of the series?

Ans. 7.

8. In an arithmetical series, the common difference is $\frac{5}{9}$ and the extremes are $14\frac{8}{9}$ and 11. What is the number of terms? *Ans.* 8.

9. In an arithmetical progression, the extremes are 14 and 86, and the number of terms is 19. What is the 11th term?

Ans. 54.

10. In an arithmetical progression, the extremes are 22 and 4, and the number of terms is 7. What is the 4th term?†

Ans. 13.

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* NOTE.—A special rule is generally given for the solution of ques tions like this, and particular rules are also given for the solution of the following questions, but if the student wants mental discipline, he had better solve these examples without the aid of any rule.

† NOTE.—When the progression is *decreasing*, we can find the last

11. One extreme is 4, the number of terms is 17, and the sum of the series is 884. What is the other extreme?

Ans. 100.

12. One extreme is 3, the number of terms is 63, and the sum of the series is 252. What is the other extreme?

Ans. 5.

GEOMETRICAL PROGRESSION.

125. A series of numbers in which each term, after the first, is found by multiplying the preceding term by a constant multiplier, is called a *geometrical progression.** When the constant multiplier is greater than unity, the progression is called an *ascending*, or *increasing* geometrical progression; and when it is less than unity, the progression is called a *descending*, or *decreasing* geometrical progression. Thus, the series 2, 6, 18, 54, in which each term, after the first, is formed by multiplying the preceding term by 3, is an ascending geometrical progression; and the series 48, 24, 12, 6, 3, in which each term, after the first, is found by multiplying the preceding term by $\frac{1}{2}$, is a descending geometrical progression. The constant multiplier is called the *common ratio.*

In Geometrical Progression there are five things to be considered, namely, the *first term*, the *last term*, the *common ratio*, the *number of terms*, and the *sum of all the terms.* Any three of these being given, the others may be found, but

term by subtracting the product of the common difference and the number of terms less one from the first term.

* Note.—Since any four successive terms of any geometrical progression, are proportional numbers, such a progression has been called a series of *continual proportionals.*

we can solve only a part of the questions that may be proposed in geometrical progression, by the aid of common arithmetic. The others may be solved by the aid of algebra.* We propose two problems in geometrical progression.

PROBLEM I.

Having given the first term, the common ratio, and the number of terms, it is required to find the last term.

It follows from the definition of geometrical progression, that the second term is equal to the first term multiplied by the ratio; that the third term is equal to the first multiplied by the second power of the ratio; that, in general, any term is equal to the first term multiplied by the ratio raised to a power whose exponent is less by 1 than the number of that term. Hence, to find the last term, *multiply the first term by the ratio raised to a power whose exponent is one less than the number of terms.*

PROBLEM II.

Having given the first term, the last term, and the common ratio, it is required to find the sum of the terms.

Let us take the progression 2, 6, 18, 54. If we multiply each term of this progression by 3, the common ratio, the sum of the several products will be equal to three times the progression, and if from this we subtract the sum of the terms in the given progression, as represented in the operation, the

* NOTE.—Arithmetical Progression and Geometrical Progression can be treated of to better advantage in Algebra. Here rules may be deduced for solving all the different questions in progression with ease and clearness. See Elements of Algebra.

remainder must be equal to twice the sum of the terms in the progression.

OPERATION.

$$
\begin{array}{rcl}
3 \text{ times the sum} & = & 6+18+54+54\times 3 \\
\text{sum} & = & 2+6+18+54 \\
\hline
2 \text{ times the sum} & = & 54\times 3-2; \\
\text{hence, } \quad \text{sum} & = & \dfrac{54\times 3-2}{2}=\dfrac{54\times 3-2}{3-1}
\end{array}
$$

Since 54 is the last term, 3 the ratio, 2 the common difference, and $3-1$ the ratio less 1, *the sum of the terms is found by multiplying the last term by the ratio, subtracting the first term from the product, and dividing the remainder by the ratio less one.*

EXAMPLES.

1. The extremes of a geometrical series are 512 and 2, and the common ratio is 4. What is the sum of the terms?

Ans. 682.

2. The extremes of a geometrical progression are 12 and 175692, and the common ratio is 11. What is the sum?

Ans. 193260.

3. The extremes of a geometrical progression are 0.3 and 937.5, and the common ratio is 5. What is the sum of the series?

Ans. 1171.875.

4. The extremes of a geometrical progression are 5 and 80 and the number of terms is 5. What is the common ratio?*

Ans. 2.

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* NOTE.—By Problem II., 80 is the product of 5 and the fourth *power* of the common ratio; hence, the fourth *root* of the quotient of 80 divided by 5 is the ratio.

5. The extremes of a geometrical progression are 1 and 15625, and the number of terms is 7. What is the common ratio?* *Ans.* 5

6. The extremes of a geometrical progression are 2011768035 and 5, and the number of terms is 10. What is the common ratio? *Ans.* 7.

7. The common ratio is 3, the number of terms is 7, and one extreme is 9; what is the other extreme? *Ans.* 6561.

8. A nobleman dying left 11 sons, to whom he bequeathed his property as follows: to the youngest he gave £1024; to the next $1\frac{1}{2}$ times as much; to the next, $1\frac{1}{2}$ times as much as he gave to the preceding son; and so on. What was the eldest son's fortune; and what was the amount of the nobleman's property?

Ans. The eldest son received £59049, and the father was worth £175099.

MISCELLANEOUS EXAMPLES.

1. How many yards of carpeting that is $\frac{3}{4}$ of a yard wide, will be required to cover a floor that is 24 feet long, and 18 feet wide; and what will the carpet cost at $1.25 per yard?

Ans. 64 yards; cost $80.

2. What is the rent of 145 A. 1 R. 32 P. of land, at £10 5s. 3d. per acre? *Ans.* £1492 13s. $7\frac{7}{25}$d.

3. A straight plank is $3\frac{1}{2}$ inches thick, and $6\frac{1}{4}$ inches broad; what length must be cut off so that the part cut off may contain $6\frac{1}{4}$ cubic feet of timber? *Ans.* $41\frac{1}{7}$ feet.

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* NOTE.—In solving this problem, we shall have to extract the sixth root of 15625. The sixth root of any number is equal to the cube root of the square root of that number.

4. If 3 pounds of tea be worth 4 pounds of coffee, and 6 pounds of coffee be worth 20 pounds of sugar, how many pounds of sugar can be had for 9 pounds of tea? *Ans.* 40.

5. In the centigrade thermometer the freezing point is zero, and the boiling point is 100°; in Fahrenheit's the freezing point is 32°, and the boiling point is 212°; what degree of the centigrade thermomer corresponds to the 68th degree of Fahrenheit? *Ans.* 20th degree.

6. What number is that to which if $\frac{3}{4}$ of $\frac{10}{7}$ of $\frac{2}{5}$ be added the sum will be 1? *Ans.* $\frac{4}{7}$.

7. A merchant having $10000, laid out $\frac{11}{16}$ of it for cloth at $2.50 per yard; how many yards did he buy?
Ans. 2750 yds.

8. A farmer bought a pile of wood 40 ft. long, 12 ft. wide, and 10 ft. high, at 4\frac{1}{4}$ per cord. What did he pay for his wood? *Ans.* $159.375.

9. A wine merchant bought $\frac{3}{5}$ of $\frac{2}{3}$ of 10 tuns of wine for $483.84; what was that per gill? *Ans.* $0.015.

10. A can dig a well in 3 days, B in 4 days, and C in 6 days; how long will it take all of them together to dig it?
Ans. 1$\frac{1}{4}$ days.

11. A person spent $\frac{2}{5}$ of his income, afterwards $\frac{2}{3}$ of $\frac{1}{6}$ of the remainder, and then found he had $400 left. What was his income? *Ans.* $750.

12. $\frac{2}{3}$ of a number exceeds $\frac{1}{4}$ of it by 60; what is the number? *Ans.* 144.

13. A owns $\frac{1}{10}$ and B $\frac{1}{6}$ of a bank; A's share is $20000 less than B's; what is the capital of the bank? *Ans.* $300000.

14. A man and boy can hoe a field of corn in 8 days; the man can hoe it alone in 12 days; how long will it take the boy to hoe it? *Ans.*

15. What number is that which, being multiplied by one-fourth of itself, the product shall be $20\frac{1}{4}$? *Ans.*

16. A father left $\frac{2}{7}$ of his estate to one of his sons, and $\frac{2}{7}$ of the remainder to the other; the rest to his wife. The difference of his sons' legacies was found to be \$400. How much did his widow receive? *Ans.* \$2500.

17. Three persons A, B and C, are to share \$50000 in the proportion of $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{7}$, respectively, but C dying, it is required to divide the whole sum properly between the other two. What were their shares? *Ans.*

18. A gentleman left his son a fortune, $\frac{1}{7}$ of which he spent the first year; $\frac{1}{3}$ of the remainder he spent the second year, after which he had \$950 left. What was his fortune at first? *Ans.* \$1662.50.

19. A person dying gave half of his estate to his wife, $\frac{1}{3}$ of the remainder to his servants, $\frac{1}{4}$ of what then remained to his oldest child, and the residue of \$7500 he distributed equally among his remaining children. What was the value of his estate? *Ans.* \$30000.

20. Bought $\frac{9}{13}$ of a tun of potash, and sold $\frac{13}{18}$ of it for \$49.50; what is the value of a ton? *Ans.* \$99.

21. If a man can earn $\$7\frac{1}{5}$ in $\frac{4}{7}$ of a week, how much can he earn in 5 weeks? *Ans.* \$63.

22. If a pole 6 feet long cast a shadow of 8 feet, what is the height of a pole that casts a shadow 200 feet? *Ans.* 150 ft.

23. Find the cube root of 228099131, and the side of a square containing 7225 sq. ft.

Ans. 611; 85 ft.

24. A general levied a contribution of £870 on four villages, containing 250, 300, 400, 500 inhabitants respectively; what must they each pay? *Ans.* £150, £180, £240, £300.

25. Find the value of $\dfrac{5\frac{4}{5}-2\frac{1}{8}}{3\frac{3}{4}+\frac{9}{20}}$ of $\dfrac{4\frac{1}{2}+5\frac{19}{25}}{4\frac{1}{20}}$ of $\dfrac{2\frac{3}{5}+1\frac{2}{3}}{7\frac{11}{24}-2\frac{1}{4}}$.

Ans. $1\frac{53}{75}$.

26. Multiply $3\frac{3}{20}$ by $15\frac{5}{7}$, and $\dfrac{2}{3\frac{3}{4}}$ by $\dfrac{2\frac{3}{4}}{3}$; and add together the sum and difference of their results. *Ans.* 99.

27. A party having a bill to pay of \$9, one of them pays for himself and three friends, \$3; how many were in the party? *Ans.* 12.

28. Add together $1\frac{3}{4}$, $2\frac{2}{3}$, and $3\frac{1}{2}$; multiply this sum by the product of these fractions; subtract from the result the difference of $2\frac{2}{3}$ and $1\frac{1}{2}$; and divide the remainder by the sum of $5\frac{1}{2}$ and $1\frac{1}{3}$ of $3\frac{3}{4}$. *Ans.* $12\frac{11}{54}$.

29. Divide $\dfrac{7(1\frac{1}{2}\text{ of }\frac{3}{14})}{\frac{1}{6}\left(\frac{3}{3\frac{1}{2}}\text{ of }7\right)}$ by $\dfrac{9}{14}$, and find the value of

$$\frac{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}{\frac{1}{2\frac{1}{2}}+\frac{1}{3\frac{1}{2}}+\frac{1}{4\frac{1}{2}}}.$$

Ans. $3\frac{1}{2}$; $1\frac{17}{88}$.

30. To $\frac{4}{15}$ of a dozen add $\frac{13}{24}$ of three hundred, and divide this sum by the difference of $3\frac{3}{4}$ of a hundred and $43\frac{3}{5}$. *Ans.* $\frac{1}{2}$.

31. A wall is to be built 15 yards long, 7 feet high, and 13 inches thick, with a doorway 6 feet high, and 4 feet wide; how

many bricks will it require, each brick containing 108 solid inches? *Ans.* 5044.

32. A and B can do a piece of work alone in 12 and 16 days, respectively; they work together at it for 3 days, when A leaves it, but B continues, and after 2 days is joined by C, and they finish it in 3 days; in what time would C do the work alone? *Ans.* 12 days.

33. A can mow $2\frac{1}{2}$ of grass in $6\frac{2}{3}$ hours, and B $2\frac{1}{6}$ acres in $5\frac{1}{3}$ hours: they mow together a field of 10 acres; in what time will they do it, and how many acres will each mow?

Ans. 12 h. 48 min.; $4\frac{4}{5}$, $5\frac{1}{5}$.

34. What is the expense of paving a rectangular court-yard, whose length is 63 feet and breadth 45 feet, it being paved with pebbles at 1s. 9d. per square yard, except a footpath, which runs the whole length, 5 feet 3 inches broad, and is paved with flag stones at 3s. per square yard?

Ans. £29 17s. $2\frac{1}{4}$d.

35. What is the present worth of $2035.75 due in 2 years $5\frac{1}{2}$ months, at $4\frac{1}{2}$ per cent.? *Ans.*

36. If 3 men can mow 7 acres of grass in 5 days of 9 hours each, in how many days of 8 hours each will 5 men mow $17\frac{1}{2}$ acres? *Ans.* $8\frac{7}{16}$.

37. If the rent of $\frac{3}{4}$ of an acre of land for 3 years, is 75 dollars, what will be the rent of 40 acres for 12 years?

Ans. $16000.

38. A man buys 27 lambs for $30, and sells 12 of them so that he loses 3 per cent. in the sale; at what price per lamb must he sell the remainder, so that he may gain $2\frac{1}{2}$ per cent. on the whole purchase? *Ans.* $.

39. Divide 240 into parts, such that $\frac{1}{4}$ of one added to $\frac{1}{10}$ of the other shall equal 36. *Ans.* 80 and 160.

40. What is the interest on \$240 for 3 years 6 months and 18 days, at 7 per cent.? *Ans.* \$59.64.

41. What is the interest on \$380 for 4 years and 4 months, at 6 per cent.? *Ans.* \$98.80.

42. Two horses have velocities of $186\frac{1}{4}$ miles in $16\frac{5}{9}$ hours, and $196\frac{7}{8}$ miles in $18\frac{2}{3}$ hours, respectively; compare their rates, and if they started together from the same point in opposite directions, how far would they be from each other in $6\frac{3}{4}$ minutes? *Ans.* 16 : 15; 2 mi. $795\frac{25}{32}$ yds.

43. By selling tea at 5s. 4d. a pound, a grocer clears $\frac{1}{8}$ of his outlay; he then raises the price to 6s.; what does he clear per cent. upon his outlay at the latter price? *Ans.* $26\frac{9}{16}$.

44. A person leaving Paddington at 13 minutes before 2, P. M., travels the first 162 miles at 27 miles per hour, the next 121 miles at $9\frac{1}{2}$ miles per hour, and the last 27 at 8 miles per hour. When will he reach his destination, Penzance?

Ans. 6 min. $17\frac{7}{19}$ sec. A. M.

45. Three persons have gained \$1320; if B were to take \$6, C ought to take 4, and D 2. What is each person's share?

Ans. B's \$660, C's \$440, D's \$220.

46. A merchant failing, owes to B \$500, and to C \$900; but he has only \$1100 to meet these demands. How much should each creditor receive? *Ans.* B \$$392\frac{6}{7}$, C \$$707\frac{1}{7}$.

47. A vintner mixed 2 gallons of wine at 14s. per gallon, with one gallon at 12s., 2 gallons at 9s., and 4 gallons at 8s. What is one gallon of the mixture worth? *Ans.* 10s.

48. A, B and C start at the same time, from the same point,

and in the same direction, round an island 73 miles in circumference; A goes at the rate of 6, B at the rate of 10, and C at the rate of 16 miles per day. In what time will they all be together again? *Ans.* 36½ days.

49. What must be the length of a plot of ground, if the breadth be 15⅝ feet, that its area may contain 46 square yards? *Ans.* 26 $\frac{62}{125}$ feet.

50. How many yards of carpeting that is ⅞ of a yard wide, will be required to carpet a room that is 35 feet long, and 18 feet wide; and what will the carpeting cost at $1.75 per yard? *Ans.* $140.

51. A and B rent a pasture for $275; A puts in 80 sheep, and B 100, but at the end of 6 months they each dispose of one half their stock, and allow C to put in 50 sheep to feed; what should A, B, and C severally pay towards the rent at the years end?

Ans. A $103.125; B $128.90625; C $42.96875.

52. A person has ¼ of a ship worth $6600, and insured for 91¼ per cent of its real value; what damage would he sustain in case that the ship should be lost? *Ans.* $82.50.

53. The circumference of a wheel is 16½ feet; how many times will it turn around in going a distance of 45 miles? *Ans.* 14400.

54. If eggs be bought at the rate of 5 for 4 pence, how must they be sold to gain 25 per cent.? *Ans.* 1 penny apiece.

55. The head of a fish was 9 inches long, its tail was as long as its head and half its body, and its body was as long as its head and tail both. What was the whole length of the fish? *Ans.* 36 inches.

56. A farmer bought a yoke of oxen, a cow and a sheep, for \$82.50; he gave for the cow 8 times as much as for the sheep, and for the oxen 3 times as much as for the cow. How much did he give for each? *Ans.*

57. At \$0.125 per yard, how many yards of cloth can be bought for \$45? *Ans.* 360.

58. If the interest on \$100 for 1 year is \$7, what is the interest on \$250 for 3 years and 6 months? *Ans.* \$61.25.

59. What is the cost of 45 bushels and 48 pounds of wheat, at \$1.25 per bushel? *Ans.*

60. How many yards of matting that is 2 feet and 6 inches wide, will cover a room that is 96 feet long and 70 feet wide? *Ans.* 896.

61. A brick is 8 inches long, 4 inches wide, and 2 inches thick. How many such bricks will be required to build a cubical cistern, open at the top, that shall contain 1000 wine gallons, if the wall is made a foot thick, and we allow that the mortar in which the bricks are laid, will constitute a fifth part of the wall? *Ans.* 3265.

62. A can do a piece of work in 3 days, and C 5 times as much in 12 days; in what time would they do it together? *Ans.* $21\frac{1}{3}$ hours.

63. A plate of gold 3 inches square, and $\frac{1}{8}$ of an inch thick, is extended by hammering so as to cover a surface of 7 square yards; find its present thickness. *Ans.* $\frac{1}{8064}$ of an inch.

64. A man can reap $302\frac{1}{2}$ square yards in an hour; in what time will 3 such men reap $2\frac{7}{9}$ acres? *Ans.* $14\frac{22}{27}$ hours.

65. If 45 bricks will pave a square yard, how many will be wanted for a space 34 feet long and 14 feet wide, if we allow for a path, two feet wide, all round? *Ans.* 1500.

66. A stationer sold quills at 11s. per thousand, by which he cleared $\frac{3}{8}$ of the money. What would he clear per cent. by selling them at 13s. 6d. per thousand? and what would he gain per cent. on his *outlay?* *Ans.*

67. There is a fraction which, when multiplied by the cube of $1\frac{1}{2}$, and divided by the square root of $1\frac{7}{9}$ produces $\frac{3}{8}$; what is the fraction? *Ans.* $\frac{4}{27}$.

68. A tradesman marks his goods 20 per cent. above the cash price; what ready money would he take for a yard of cloth marked \$5.40? *Ans.* \$4.50.

69. At 27 cents per square yard, what is the cost of painting a room which is 24 yards round, and 10 feet 4 in. high? *Ans.* \$22.32.

70. A farmer gave for a horse a note of \$156, due in 8 months, at $4\frac{1}{2}$ per cent., and sold him at once for \$180; what is his gain per cent.? *Ans.* $18\frac{11}{13}$.

71. If five men can reap a field 800 feet long and 700 feet wide in $3\frac{1}{2}$ days of 14 hours each; in how many days of 12 hours each will 7 men reap a field 1800 feet long and 960 wide? *Ans.* 9 days.

72. What will it cost to carpet a room that is 21 feet long and 18 feet wide, if the carpeting is 2 feet wide, and costs \$1.75 per yard? *Ans.* \$109.25.

73. A does $\frac{5}{9}$ of a piece of work in 10 days, when B comes to help him, and they take 3 days more to finish it; in what time would they have done the whole, each separately, or both together? *Ans.* 18, $10\frac{4}{5}$, $6\frac{3}{4}$ days.

74. Out of a cask of wine, of which a fifth part had leaked away, 10 gallons were drawn, and then it was two-thirds full; how much did it hold? *Ans.* 75 gallons.

75. A cistern can be filled in one half of an hour by a pipe A, and emptied in 20 minutes by another pipe B: after A had been opened 20 minutes, B is also opened for 12 minutes, when A is closed, and B remains open for 5 minutes more, and now there are 13 gallons in the cistern; how much would it contain when full? *Ans.* 60 gallons.

76. A can do a piece of work in 10 days; but after he has been upon it 4 days, B is sent to help him, and they finish it together in 2 days. In what time would B have done the whole? *Ans.* 5 days.

77. According to the census of 1840 there were in the state of New York 44452 white persons over 20 years of age, who could neither read nor write; and in the state of North Carolina there were 56609 such persons. By the same census the population of New York was 2428921, and that of North Carolina was 753419. What was the percentage of illiterate persons in each of these states?

Ans. New York 1.83+; N. C. 7.5+.

78. A gamester at one sitting lost $\frac{1}{2}$ of his money, and then won 10 shillings; at a second sitting he lost $\frac{1}{3}$ of the remainder, and then won 3 shillings; he then had 63 shillings left. How much money had he at first?

Ans. 100 shillings.

79. A and B have the same income. A lays by a fifth part of his; but B, by spending $80 more than A, finds himself at the end of 4 years $220 in debt. What was their income? *Ans.* $125.

80. A can build a wall in 8 days, which A and B can build in 5 days by working together: how long would B take to do it alone? and how long after B has begun should

A begin, so that, by finishing it together, they may each have built half of the wall? *Ans.* $13\frac{1}{3}$ days; $2\frac{2}{3}$ days.

81. A cistern can be filled in 15 minutes by 2 pipes, A and B, running together. After A has been running by itself for 5 minutes, B is also turned on, and the cistern is filled in 13 minutes more; in what time would it be filled by each pipe separately? *Ans.* $37\frac{1}{2}$ minutes; 25 minutes.

82. A man could reap a field by himself in 20 hours; but with his son's help for 6 hours, he could do it in 16 hours; how long would the son be in reaping the field by himself? *Ans.* 30 hours.

83 A and B start to run a race; at the end of 5 minutes, when A has run 900 yards, and has outstripped B by 75 yards, he falls; but, though he loses ground by the accident, and for the rest of the course makes 20 yards a minute less than before, he comes in only half a minute behind B. How long did the race last? *Ans.* 36 minutes.

84. A can do a piece of work in 10 days, which B can do in 8 days. After A has been at work upon it 3 days, B comes to help him; in what time will they finish it? *Ans.* $3\frac{1}{9}$ days.

85. A met two beggars, B and C, and, having a certain sum in his pocket, gave $\frac{3}{28}$ of it to B, and $\frac{3}{5}$ of the remainder to C. A had now 20 cents left; what had he at first? *Ans.* \$0.56.

86. How many acres will supply 53 horses with hay and oats, if each horse consumes annually the produce of 5 A. 3 R. 26 P.? *Ans.* 313 A. 1 R. 18 P.

87. How many yards of carpet, 25 inches wide, will it take to cover a floor that is 19 feet 7 inches long and 18 feet 9 inches wide? *Ans.* 58 yards 3 feet 2 inches.

88. A and B can reap a field together in 12 hours, A and C in 16 hours, and A by himself in 20 hours. In what time time could B and C together reap the field, and in what time could A B and C together reap it?

Ans. $21\frac{9}{11}$ hours; $10\frac{10}{23}$ hours.

89. What is the area of a rectangular field, the diagonal of which is 60 rods, and the width of which is 60 rods, and the width of which is 36 rods? *Ans.* $10\frac{4}{5}$ acres.

90. What will it cost to enclose 35 A. 0 R. 25 P., in the form of a square, at the rate of \$1.50 per rod for the fencing?

Ans. \$450.

91. A boy, selling oranges, sells half of his stock and one more to A, half of what remains and two more to B, and three that still remained to C; how many had he at first?

Ans. 22.

92. A, after spending \$10 less than the third of his yearly income, found that he had \$45 more than half of it remaining; what was his income? *Ans.* \$210.

93. A can do a piece of work in $10\frac{1}{2}$ days, which A and B can do together in $5\frac{3}{5}$ days; how many days would B require to do it alone? *Ans.* 12.

94. A can correct 70 pages for the press in $1\frac{1}{2}$ hours, and B can correct 150 pages in $2\frac{1}{4}$ hours. How long will they be in correcting 425 pages jointly? *Ans.* $3\frac{3}{4}$ hours.

95. A hare starts 50 leaps before a grey-hound, and takes 4 leaps, while the hound takes 3; but two of the hound's leaps are equal to 3 of the hare's. How many leaps must the hound take to overtake the hare? *Ans.*

96. A person paid \$17.85 for 21 pounds of tea and 60

pounds of coffee. The tea cost 31 cents a pound more than the coffee. What was the price of each per pound?

Ans. Tea 45 cents; Coffee 14 cents

97. A man paid $5625 for a farm, and the price per acre was equal to the number of acres; how many acres did the farm contain? *Ans.* $75.

98. A stationer sold quills at $2 per thousand, and gained $\frac{1}{4}$ of the cost; but quills growing scarce, he raised the price to $2.16. What did he gain per cent. by the last sale?

Ans. 35.

99. A merchant sold tea at 54 cents per pound, and lost 10 per cent. He afterwards sold a parcel of the same kind of tea for $53.55, and gained 5 per cent. How many pounds did he sell? *Ans.* 85.

100. A boy purchased a certain number of oranges, at the rate of 2 for a cent, and as many more at the rate of 3 for a cent; and then sold them all at the rate of 5 for 2 cents, and lost 4 cents by the transaction. How many did he buy?

Ans. 240.

101. What will it cost to have a walk around a square park that contains 10 acres, if the walk is 4 feet wide, and the piece of paving is $6\frac{1}{4}$ cents per square foot? *Ans.* $656

APPENDIX.

MENSURATION.

Geometry is a science which has for its object the measurement of extension, and the application of the truths of geometry to the measurement of surfaces and solids is called *Mensuration.*

The rules for the measurement of surfaces and solids, are generally stated in geometry as being so many distinct propositions, and a rigorous demonstration of each is given. The student, therefore, who wishes to know the reason for the several rules here given, must consult a treatise on geometry.

DEFINITIONS OF GEOMETRICAL TERMS.

1. A *straight line* is one which does not change its direction at any point.

2. Two lines are *parallel* when they have the same direction.

3. A *curve line* is one that is constantly changing its direction.

4. A *plane* is a surface, such that if any two points be taken in it, and connected by a straight line, this straight line will lie wholly in the surface.

5. A *plane figure* is any portion of a plane which is bounded by lines either straight or curved.

6. A *polygon* is any portion of a plane included by straight lines. The sum of these lines is called the *perimeter* of the polygon. The polygon of three sides is called a *triangle;* that of four sides is called a *quadrilateral;* that of five sides is called a *pentagon;* that of six sides is called a *hexagon;* and so on.

7. An *equilateral polygon* is one whose boundary lines are equal each to each.

8. A *parallelogram* is a quadrilateral whose opposite sides are parallel.

9. A *trapezoid* is a quadrilateral which has only two of its sides parallel.

10. The *base* of a figure is the side on which it is supposed to stand.

11. The *altitude* of a triangle is the perpendicular let fall from the vertex of one of its angles to the opposite side taken as the base.

12. The *altitude* of a paralellogram is the perpendicular which joins two opposite sides taken as bases.

13. The *altitude* of a trapezoid is the perpendicular which joins the two parallel sides.

14. The *circumference* of a circle is a curved line, all the points of which are situated in the same plane, and at an equal distance from a point within, called the *centre.* The circle is that portion of the plane included by the circumference.

15. The radius of a circle is any straight line drawn from the *entre to* the circumference. Any straight line passing through

the centre, and limited by the circumference, is called a *diameter* of the circle.

16. Any portion of the circumference is called an *arc*. The line which joins the extremities of an arc is called a *chord*.

17. A *segment* of a circle is that portion of it included by an arc and its chord.

18. A *sector* of a circle is that portion of it included by two radii and their intercepted arc.

19. A *prism* is a solid the ends of which are terminated by two equal polygons, with their planes parallel and its faces are equal parallelograms. The equal and parallel polygons are called the *bases* of the prism. The straight line formed by the intersection of any two adjacent faces is called the *edge* of the prism. When the edge of a prism is perpendicular to the planes of the bases, the prism is called a *right prism*. The sum of the equal parallelograms is called the convex surface of the prism.

20. The altitude of a prism is the perpendicular distance between its two bases.

21. A *cylinder* is a round body, the ends of which are equal circles, whose planes are parallel. The equal circles are called bases of the cylinder, and the perpendicular distance between the bases, is called the *altitude* of the cylinder.

22. A *pyramid* is a solid bounded by a polygon and plane triangles. The polygon is called the *base* of the pyramid, and the plain triangles, taken together, form the *convex surface* of the pyramid. The common vertex of the triangles is called the *vertex* of the pyramid. The line drawn from the vertex perpendicular to the plane of the base is called the *altitude* of the

prism. The line drawn from the vertex perpendicular to either side of its base, is called the *slant height.*

23. A *cone* is a solid which has a circle for its base, and which tapers uniformly to a point, which point is called the *vertex* of the cone. A line drawn from the vertex to the centre of the base, is called the *altitude* of the cone. The *slant height* of a cone is the line drawn from its vertex to the circumference of its base.

24. The *frustum of a pyramid* or *cone*, is the part that remains after cutting off the top by passing a plane parallel to its base.

25. A *sphere* is a solid, every point of whose surface is equally distant from a point within called the centre. A line drawn from the centre to the circumference is called the *radius* of the sphere. A line passing through the centre of the sphere and terminated by the surface, is called a *diameter* of the sphere.

RULES FOR THE MENSURATION OF SURFACES AND SOLIDS.

MENSURATION OF SURFACES.

RULE I.

To find the area of a parallelogram.

Multiply the base by the altitude.

RULE II.

To find the area of a triangle.

Multiply the base by one half of the altitude. If the three

sides are given, find their half sum, and from this half sum subtract each of the three sides separately; and then multiply together the half sum and the three remainders; the square root of the product will be the required area.

RULE III.

To find the area of a trapezoid.

Multiply the half sum of the two parallel sides by the altitude, and the product will be the required area.

RULE IV.

To find the circumference of a circle.

Multiply the diameter by 3.1416, *and the product will be the circumference.*

RULE V.

To find the area of a circle.

Multiply the square of the diameter by 0.7854, *and the product will be the area. Or, multiply the circumference by half of the radius, and the product will be the area.*

RULE VI.

Give the altitude of a prism, and the perimeter of its base, to find the convex surface.

Multiply the perimeter of its base by its altitude, and the product will be the convex surface.

RULE VII.

To find the area of a regular polygon.

Multiply the square of the side of the polygon by the mul-

tiplier set opposite to its name in the following table, and the product will be the area.

Number of Sides.	Names.	
3	Triangle . .	0.4330127
4	Square . . .	1.0000000
5	Pentagon . .	1.7204774
6	Hexagon . .	2.5980762
7	Heptagon . .	3.6339124
8	Octagon . .	4.8284271
9	Nonagon . .	6.1818242
10	Decagon . .	7.6942088
11	Undecagon . .	9.3656399
12	Dodecagon . .	11.1961524

RULE VIII.

To find the surface of a sphere.

Multiply the square of its diameter by 3.1416, *and the product will be its surface.*

SOLIDS.

RULE IX.

To find the solidity of any prism or cylinder.

Multiply the area of the base by its altitude, and the pro duct will be solidity.

RULE X.

To find the solidity of a pyramid, or cone.

Multiply the area of its base by one third of its altitude, and the product will be its solidity.

RULE XI.

To find the solidity of the frustum of a pyramid, or that of a cone.

To the sum of the area of the upper and lower bases of the frustum, add a mean proportional between them, and multiply this sum by one-third of the altitude, and the product will be the solidity.*

RULE XII.

To find the solidity of a sphere.

Multiply the cube of its diameter by 0.5236, *and the product will be the solidity.*

EXAMPLES.

1. What is the area of a triangle whose base and altitude are respectively 60, and 48? *Ans.* 1440.

2. How many square yards are there in a triangle whose sides are 30 feet, 40 feet, and 50 feet, respectively? *Ans.* $66\frac{2}{3}$.

3. How many square feet in a plank of the form of a trapezoid, whose length is $12\frac{1}{2}$ feet, and the breadth of one end is 15 inches, and at the less 11 inches? *Ans.* $13\frac{13}{24}$.

4. What is the circumference of a circle whose diameter is 25 feet? *Ans.* 78.54 feet.

5. What is the area of a circle whose diameter is 10 feet? *Ans.* 78.54 sq. feet.

6. The altitude of a prism is 20 feet, and the perimeter of its base is 12 feet. What is its convex surface? *Ans.*

* NOTE.—A mean proportional between two numbers is equal to the square root of their product.

7. What is the solidity of a cylinder whose length is 20 feet, and circumference $5\frac{1}{2}$ feet? *Ans.* 48.1459 feet.

8. What is the solidity of a cone, its height being $10\frac{1}{2}$ feet, and the circumference of its base 9 feet?

Ans. 22.56 cubic feet.

9. If a cask, which is two equal conic frustums joined together at their bases, have its bung diameter 28 inches, the head diameter 20 inches, and length 40 inches; how many gallons of wine will it hold? *Ans.* 79.0613.

10. What is the solidity of a sphere whose diameter is 12 inches? *Ans.* 904.78 cu. in.

A TABLE

OF

SQUARE AND CUBE ROOTS.

No.	Sq. Root.	Cube Root.	No.	Sq. Root.	Cube Root.	No.	Sq. Root.	Cube Root.
1	1·0000000	1·000000	40	6·3245553	3·419952	79	8·8881944	4·290841
2	1·4142136	1·259921	41	6·4031242	3·448217	80	8·9442719	4·308870
3	1·7320508	1·442250	42	6·4807407	3·476027	81	9·0000000	4·326749
4	2·0000000	1·587401	43	6·5574385	3·503398	82	9·0553851	4·344481
5	2·2360680	1·709976	44	6·6332496	3·530348	83	9·1104336	4·362071
6	2·4494897	1·817121	45	6·7082039	3·556893	84	9·1651514	4·379519
7	2·6457513	1·912931	46	6·7823300	3·583048	85	9·2195445	4·396830
8	2·8284271	2·000000	47	6·8556546	3·608826	86	9·2736185	4·414005
9	3·0000000	2·080084	48	6·9282032	3·634241	87	9·3273791	4·431047
10	3·1622777	2·154435	49	7·0000000	3·659306	88	9·3808315	4·447960
11	3·3166248	2·223980	50	7·0710678	3·684031	89	9·4339811	4·464745
12	3·4641016	2·289428	51	7·1414284	3·708430	90	9·4868330	4·481405
13	3·6055513	2·351335	52	7·2111026	3·732511	91	9·5393920	4·497941
14	3·7416574	2·410142	53	7·2801099	3·756286	92	9·5916630	4·514357
15	3·8729833	2·466212	54	7·3484692	3·779763	93	9·6436508	4·530655
16	4·0000000	2·519842	55	7·4161985	3·802953	94	9·6953597	4·546836
17	4·1231056	2·571282	56	7·4833148	3·825862	95	9·7467943	4·562903
18	4·2426407	2·620741	57	7·5498344	3·848501	96	9·7979590	4·578857
19	4·3588989	2·668402	58	7·6157731	3·870877	97	9·8488578	4·594701
20	4·4721360	2·714418	59	7·6811457	3·892996	98	9·8994949	4·610436
21	4·5825757	2·758924	60	7·7459667	3·914867	99	9·9498744	4·626065
22	4·6904158	2·802039	61	7·8102497	3·936497	100	10·0000000	4·641589
23	4·7958315	2·843867	62	7·8740079	3·957892	101	10·0498756	4·657010
24	4·8989795	2·884499	63	7·9372539	3·979057	102	10·0995049	4·672329
25	5·0000000	2·924018	64	8·0000000	4·000000	103	10·1488916	4·687548
26	5·0990195	2·962496	65	8·0622577	4·020726	104	10·1980390	4·702669
27	5·1961524	3·000000	66	8·1240384	4·041240	105	10·2469508	4·717694
28	5·2915026	3·036589	67	8·1853528	4·061548	106	10·2956301	4·732624
29	5·3851648	3·072317	68	8·2462113	4·081656	107	10·3440804	4·747459
30	5·4772256	3·107232	69	8·3066239	4·101566	108	10·3923048	4·762203
31	5·5677644	3·141381	70	8·3666003	4·121285	109	10·4403065	4·776856
32	5·6568542	3·174802	71	8·4261498	4·140818	110	10·4880885	4·791420
33	5·7445626	3·207534	72	8·4852814	4·160168	111	10·5356538	4·805896
34	5·8309519	3·239612	73	8·5440037	4·179339	112	10·5830052	4·820284
35	5·9160798	3·271066	74	8·6023253	4·198336	113	10·6301458	4·834588
36	6·0000000	3·301927	75	8·6602540	4·217163	114	10·6770783	4·848808
37	6·0827625	3·332222	76	8·7177979	4·235824	115	10·7238053	4·862944
38	6·1644140	3·361975	77	8·7749644	4·254321	116	10·7703296	4·876999
39	6·2449980	3·391211	78	8·8317609	4·272659	117	10·8166538	4·890973

No.	Sq. Root.	Cube Root.	No.	Sq. Root.	Cube Root.	No.	Sq. Root.	Cube Root.
118	10·8627805	4·904868	174	13·1909060	5·582770	230	15·1657509	6·126925
119	10·9087121	4·918685	175	13·2287566	5·593445	231	15·1986842	6·135792
120	10·9544512	4·932424	176	13·2664992	5·604079	232	15·2315462	6·144634
121	11·0000000	4·946088	177	13·3041347	5·614673	233	15·2643375	6·153449
122	11·0453610	4·959675	178	13·3416641	5·625226	234	15·2970585	6·162239
123	11·0905365	4·973190	179	13·3790882	5·635741	235	15·3297097	6·171005
124	11·1355287	4·986631	180	13·4164079	5·646216	236	15·3622915	6·179747
125	11·1803399	5·000000	181	13·4536240	5·656651	237	15·3948043	6·188463
126	11·2249722	5·013298	182	13·4907376	5·667051	238	15·4272486	6·197154
127	11·2694277	5·026526	183	13·5277493	5·677411	239	15·4596248	6·205821
128	11·3137085	5·039684	184	13·5646600	5·687734	240	15·4919334	6·214464
129	11·3578167	5·052774	185	13·6014705	5·698019	241	15·5241747	6·223084
130	11·4017543	5·065797	186	13·6381817	5·708267	242	15·5563492	6·231679
131	11·4455231	5·078753	187	13·6747943	5·718479	243	15·5884573	6·240251
132	11·4891253	5·091643	188	13·7113092	5·728654	244	15·6204994	6·248800
133	11·5325626	5·104469	189	13·7477271	5·738794	245	15·6524758	6·257324
134	11·5758369	5·117230	190	13·7840488	5·748897	246	15·6843871	6·265826
135	11·6189500	5·129928	191	13·8202750	5·758965	247	15·7162336	6·274305
136	11·6619038	5·142563	192	13·8564065	5·768998	248	15·7480157	6·282760
137	11·7046999	5·155137	193	13·8924440	5·778996	249	15·7797338	6·291194
138	11·7473444	5·167649	194	13·9283883	5·788960	250	15·8113883	6·299604
139	11·7898261	5·180101	195	13·9642400	5·798890	251	15·8429795	6·307993
140	11·8321596	5·192494	196	14·0000000	5·808786	252	15·8745079	6·316359
141	11·8743421	5·204828	197	14·0356688	5·818648	253	15·9059737	6·324704
142	11·9163753	5·217103	198	14·0712473	5·828476	254	15·9373775	6·333026
143	11·9582607	5·229321	199	14·1067360	5·838272	255	15·9687194	6·341326
144	12·0000000	5·241483	200	14·1421356	5·848035	256	16·0000000	6·349604
145	12·0415946	5·253588	201	14·1774469	5,857766	257	16·0312195	6·357861
146	12·0830460	5·265637	202	14·2126704	5·867464	258	16·0623784	6·366095
147	12·1243557	5·277632	203	14·2478068	5·877130	259	16·0934769	6·374311
148	12·1655251	5·289572	204	14·2828569	5·886765	260	16·1245155	6·382504
149	12·2065556	5·301459	205	14·3178211	5·896368	261	16·1554944	6·390676
150	12·2474487	5·313293	206	14·3527001	5·905941	262	16·1864141	6·398828
151	12·2882056	5·325074	207	14·3874946	5·915483	263	16·2172747	6·406958
152	12·3288280	5·336803	208	14·4222051	5·924993	264	16·2480768	6·415068
153	12·3693169	5·348481	209	14·4568323	5·934473	265	16·2788206	6·423158
154	12·4096736	5·360108	210	14·4913767	5·943921	266	16·3095064	6·431228
155	12·4498996	5·371685	211	14·5258390	5·953341	267	16·3401346	6·439277
156	12·4899960	5·383213	212	14·5602198	5·962731	268	16·3707055	6·447305
157	12·5299641	5·394691	213	14·5945195	5·972091	269	16·4012195	6·455315
158	12·5698051	5·406120	214	14·6287388	5·981426	270	16·4316767	6·463304
159	12·6095202	5·417501	215	14·6628783	5·990727	271	16·4620776	6·471274
160	12·6491106	5·428835	216	14·6969385	6·000000	272	16·4924225	6·479224
161	12·6885775	5·440122	217	14·7309199	6·009244	273	16·5227116	6·487154
162	12·7279221	5·451362	218	·14·7648231	6·018463	274	16·5529454	6·495065
163	12·7671453	5·462556	219	14·7986486	6·027650	275	16·5831240	6·502956
164	12·8062485	5·473704	220	14·8323970	6·036811	276	16·6132477	6·510830
165	12·8452326	5·484806	221	14·8660687	6·045943	277	16·6433170	6·518684
166	12·8840987	5·495865	222	14·8996644	6·055048	278	16·6733320	6·526519
167	12·9228480	5·506879	223	14·9331845	6·064126	279	16·7032931	6·534335
168	12·9614814	5·517848	224	14·9666295	6·073178	280	16·7332005	6·542133
169	13·0000000	5·528775	225	15·0000000	6·082201	281	16·7630546	6·549912
170	13·0384048	5·539658	226	15·0332964	6·091199	282	16·7928556	6·557672
171	13·0766968	5·550499	227	15·0665192	6·100170	283	16·8226038	6·565415
172	13·1148770	5·561298	228	15·0996689	6·109115	284	16·8522995	6·573139
173	13·1529464	5·572055	229	15·1327460	6·118033	285	16·8819430	6·580844

No.	Sq. Root.	Cube Root.	No.	Sq. Root.	Cube Root.	No.	Sq. Root.	Cube Root.
286	16·9115345	6·588532	342	18·4932420	6·993191	398	19·9499373	7·355762
287	16·9410743	6·596202	343	18·5202592	7·000000	399	19·9749844	7·361918
288	16·9705627	6·603854	344	18·5472370	7·006796	400	20·0000000	7·368063
289	17·0000000	6·611489	345	18·5741756	7·013579	401	20·0249844	7·374198
290	17·0293864	6·619106	346	18·6010752	7·020349	402	20·0499377	7·380322
291	17·0587221	6·626705	347	18·6279360	7·027106	403	20·0748599	7·386437
292	17·0880075	6·634287	348	18·6547581	7·033850	404	20·0997512	7·392542
293	17·1172428	6·641852	349	18·6815417	7·040581	405	20·1246118	7·398636
294	17·1464282	6·649399	350	18·7082869	7·047298	406	20·1494417	7·404720
295	17·1755640	6·656930	351	18·7349940	7·054004	407	20·1742410	7·410795
296	17·2046505	6·664444	352	18·7616630	7·060696	408	20·1990099	7·416859
297	17·2336879	6·671940	353	18·7882942	7·067376	409	20·2237484	7·422914
298	17·2626762	6·679420	354	18·8148877	7·074044	410	20·2484567	7·428959
299	17·2916165	6·686882	355	18·8414437	7·080699	411	20·2731349	7·434994
300	17·3205081	6·694329	356	18·8679623	7·087341	412	20·2977831	7·441019
301	17·3493516	6·701759	357	18·8944436	7·093971	413	20·3224014	7·447034
302	17·3781472	6·709173	358	18·9208879	7·100588	414	20·3469899	7·453040
303	17·4068952	6·716570	359	18·9472953	7·107194	415	20·3715488	7·459036
304	17·4355958	6·723951	360	18·9736660	7·113786	416	20·3960781	7·465022
305	17·4642492	6·731316	361	19·0000000	7·120367	417	20·4205779	7·470999
306	17·4928557	6·738665	362	19·0262976	7·126936	418	20·4450483	7·476966
307	17·5214155	6·745997	363	19·0525589	7·133492	419	20·4694895	7·482924
308	17·5499288	6·753313	364	19·0787840	7·140037	420	20·4939015	7·488872
309	17·5783958	6·760614	365	19·1049732	7·146569	421	20·5182845	7·494811
310	17·6068169	6·767899	366	19·1311265	7·153090	422	20·5426386	7·500741
311	17·6351921	6·775169	367	19·1572441	7·159599	423	20·5669638	7·506661
312	17·6635217	6·782423	368	19·1833261	7·166096	424	20·5912603	7·512571
313	17·6918060	6·789661	369	19·2093727	7·172580	425	20·6155281	7·518473
314	17·7200451	6·796884	370	19·2353841	7·179054	426	20·6397674	7·524365
315	17·7482393	6·804092	371	19·2613603	7·185516	427	20·6639783	7·530248
316	17·7763888	6·811284	372	19·2873015	7·191966	428	20·6881609	7·536121
317	17·8044938	6·818462	373	19·3132079	7·198405	429	20·7123152	7·541986
318	17·8325545	6·825624	374	19·3390796	7·204832	430	20·7364414	7·547842
319	17·8605711	6·832771	375	19·3649167	7·211248	431	20·7605395	7·553688
320	17·8885438	6·839904	376	19·3907194	7·217652	432	20·7846097	7·559526
321	17·9164729	6·847021	377	19·4164878	7·224045	433	20·8086520	7·565355
322	17·9443584	6·854124	378	19·4422221	7·230427	434	20·8326667	7·571174
323	17·9722008	6·861212	379	19·4679223	7·236797	435	20·8566536	7·576985
324	18·0000000	6·868285	380	19·4935887	7·243156	436	20·8806130	7·582786
325	18·0277564	6·875344	381	19·5192213	7·249504	437	20·9045450	7·588579
326	18·0554701	6·882388	382	19·5448203	7·255841	438	20·9284495	7·594363
327	18·0831413	6·889419	383	19·5703858	7·262167	439	20·9523268	7·600138
328	18·1107703	6·896435	384	19·5959179	7·268482	440	20·9761770	7·605905
329	18·1383571	6·903436	385	19·6214169	7·274786	441	21·0000000	7·611662
330	18·1659021	6·910423	386	19·6468827	7·281079	442	21·0237960	7·617412
331	18·1934054	6·917396	387	19·6723156	7·287362	443	21·0475652	7·623152
332	18·2208672	6·924355	388	19·6977156	7·293633	444	21·0713075	7·628884
333	18·2482876	6·931301	389	19·7230829	7·299894	445	21·0950231	7·634607
334	18·2756669	6·938232	390	19·7484177	7·306143	446	21·1187121	7·640321
335	18·3030052	6·945149	391	19·7737199	7·312383	447	21·1423745	7·646027
336	18·3303028	6·952053	392	19·7989899	7·318611	448	21·1660105	7·651725
337	18·3575598	6·958943	393	19·8242276	7·324829	449	21·1896201	7·657414
338	18·3847763	6·965819	394	19·8494332	7·330937	450	21·2132034	7·663094
339	18·4119526	6·972683	395	19·8746069	7·337234	451	21·2367606	7·668766
340	18·4390889	6·979532	396	19·8997487	7·343420	452	21·2602916	7·674430
341	18·4661853	6·986368	397	19·9248588	7·349597	453	21·2837967	7·680086

No.	Sq. Root.	Cube Root.	No.	Sq. Root.	Cube Root.	No.	Sq. Root.	Cube Root.
454	21·3072758	7·685733	510	22·5831796	7·989570	566	23·7907545	8·271904
455	21·3307290	7·691372	511	22·6053091	7·994788	567	23·8117618	8·286773
456	21·3541565	7·697002	512	22·6274170	8·000000	568	23·8327506	8·218635
457	21·3775583	7·702625	513	22·6495033	8·005205	569	23·8537209	8·286493
458	21·4009346	7·708239	514	22·6715681	8·010403	570	23·8746728	8·291344
459	21·4242853	7·713845	515	22·6936114	8·015595	571	23·8956063	8·296190
460	21·4476106	7·719442	516	22·7156334	8·020779	572	23·9165215	8·301030
461	21·4709106	7·725032	517	22·7376340	8·025957	573	23·9374184	8·305865
462	21·4941853	7·730614	518	22·7596134	8·031129	574	23·9582971	8·310694
463	21·5174348	7·736188	519	22·7815715	8·036293	575	23·9791576	8·315517
464	21·5406592	7·741753	520	22·8035085	8·041451	576	24·0000000	8·320335
465	21·5638587	7·747311	521	22·8254244	8·046603	577	24·0208243	8·325147
466	21·5870331	7·752861	522	22·8473193	8·051748	578	24·0416306	8·329954
467	21·6101828	7·758402	523	22·8691933	8·056886	579	24·0624188	8·334755
468	21·6333077	7·763936	524	22·8910463	8·062018	580	24·0831892	8·339551
469	21·6564078	7·769462	525	22·9128785	8·067143	581	24·1039416	8·344341
470	21·6794834	7·774980	526	22·9346899	8·072262	582	24·1246762	8·349126
471	21·7025344	7·780490	527	22·9564806	8·077374	583	24·1453929	8·353905
472	21·7255610	7·785993	528	22·9782506	8·082480	584	24·1660919	8·358678
473	21·7485632	7·791487	529	23·0000000	8·087579	585	24·1867732	8·363446
474	21·7715411	7·796974	530	23·0217289	8·092672	586	24·2074369	8·368209
475	21·7944947	7·802454	531	23·0434372	8·097759	587	24·2280829	8·372967
476	21·8174242	7·807925	532	23·0651252	8·102839	588	24·2487113	8·377719
477	21·8403297	7·813389	533	23·0867928	8·107913	589	24·2693222	8·382465
478	21·8632111	7·818846	534	23·1084400	8·112980	590	24·2899156	8·387206
479	21·8860686	7·824294	535	23·1300670	8·118041	591	24·3104916	8·391942
480	21·9089023	7·829735	536	23·1516738	8·123096	592	24·3310501	8·396673
481	21·9317122	7·835169	537	23·1732605	8·128145	593	24·3515913	8·401398
482	21·9544984	7·840595	538	23·1948270	8·133187	594	24·3721152	8·406118
483	21·9772610	7·846013	539	23·2163735	8·138223	595	24·3926218	8·410833
484	22·0000000	7·851424	540	23·2379001	8·143253	596	24·4131112	8·415542
485	22·0227155	7·856828	541	23·2594067	8·148276	597	24·4335834	8·420246
486	22·0454077	7·862224	542	23·2808935	8·153294	598	24·4540385	8·424945
487	22·0680765	7·867613	543	23·3023604	8·168305	599	24·4744765	8·429638
488	22·0907220	7·872994	544	23·3238076	8·163310	600	24·4948974	8·434327
489	22·1133444	7·878368	545	23·3452351	8·168309	601	24·5153013	8·439010
490	22·1359436	7·883735	546	23·3666429	8·173302	602	24·5356883	8·443688
491	22·1585198	7·889095	547	23·3880311	8·178289	603	24·5560583	8·448360
492	22·1810730	7·894447	548	23·4093998	8·183269	604	24·5764115	8·453028
493	22·2036033	7·899792	549	23·4307490	8·188244	605	24·5967478	8·457691
494	22·2261108	7·905129	550	23·4520788	8·193213	606	24·6170673	8·462348
495	22·2485955	7·910460	551	23·4733892	8·198175	607	24·6373700	8·467000
496	22·2710575	7·915783	552	23·4946802	8·203132	608	24·6576560	8·471647
497	22·2934968	7·921100	553	23·5159520	8·208082	609	24·6779254	8·476289
498	22·3159136	7·926408	554	23·5372046	8·213027	610	24·6981781	8·480926
499	22·3383079	7·931710	555	23·5584380	8·217966	611	24·7184142	8·485558
500	22·3606798	7·937005	556	23·5796522	8·222898	612	24·7386338	8·490185
501	22·3830293	7·942293	557	23·6008474	8·227825	613	24·7588368	8·494806
502	22·4053565	7·947574	558	23·6220236	8·232746	614	24·7790234	8·499423
503	22·4276615	7·952848	559	23·6431808	8·237661	615	24·7991935	8·504035
504	22·4499443	7·958114	560	23·6643191	8·242571	616	24·8193473	8·508642
505	22·4722051	7·963374	561	23·6854386	8·247474	617	24·8394847	8·513243
506	22·4944438	7·968627	562	23·7065392	8·252371	618	24·8596058	8·517840
507	22·5166605	7·973873	563	23·7276210	8·257263	619	24·8797106	8·522432
508	22·5388553	7·979112	564	23·7486842	8·262149	620	24·8997992	8·527019
509	22·5610283	7·984344	565	23·7697286	8·267029	621	24·9198716	8·531601

No.	Sq. Root.	Cube Root.	No.	Sq. Root.	Cube Root.	No.	Sq. Root.	Cube Root.
622	24·9399278	8·536178	678	26·0384331	8·785029	734	27·0924344	9·020529
623	24·9599679	8·540750	679	26·0576284	8·789346	735	27·1108834	9·024624
624	24·9799920	8·545317	680	26·0768096	8·793659	736	27·1293199	9·028715
625	25·0000000	8·549879	681	26·0959767	8·797968	737	27·1477439	9·032802
626	25·0199920	8·554437	682	26·1151297	8·802272	738	27·1661554	9·036886
627	25·0399681	8·558990	683	26·1342687	8·806572	739	27·1845544	9·040965
628	25·0599282	8·563538	684	26·1533937	8·810868	740	27·2029410	9·045041
629	25·0798724	8·568081	685	26·1725047	8·815160	741	27·2213152	9·049114
630	25·0998008	8·572619	686	26·1916017	8·819447	742	27·2396769	9·053183
631	25·1197134	8·577152	687	26·2106848	8·823731	743	27·2580263	9·057248
632	25·1396102	8·581681	688	26·2297541	8·828009	744	27·2763634	9·061310
633	25·1594913	8·586205	689	26·2488095	8·832285	745	27·2946881	9·065367
634	25·1793566	8·590724	690	26·2678511	8·836556	746	27·3130006	9·069422
635	25·1992063	8·595238	691	26·2868789	8·840823	747	27·3313007	9·073473
636	25·2190404	8·599747	692	26·3058929	8·845085	748	27·3495887	9·077520
637	25·2388589	8·604252	693	26·3248932	8·849344	749	27·3678644	9·081563
638	25·2586619	8·608753	694	26·3438797	8·853598	750	27·3861279	9·085603
639	25·2784493	8·613248	695	26·3628527	8·857849	751	27·4043792	9·089639
640	25·2982213	8·617739	696	26·3818119	8·862095	752	27·4226184	9·093672
641	25·3179778	8·622225	697	26·4007576	8·866337	753	27·4408455	9·097701
642	25·3377189	8·626706	698	26·4196896	8·870576	754	27·4590604	9·101726
643	25·3574447	8·631183	699	26·4386081	8·874810	755	27·4772633	9·105748
644	25·3771551	8·635655	700	26·4575131	8·879040	756	27·4954542	9·109766
645	25·3968502	8·640123	701	26·4764046	8·883266	757	27·5136330	9·113781
646	25·4165301	8·644585	702	26·4952826	8·887488	758	27·5317998	9·117793
647	25·4361947	8·649044	703	26·5141472	8·891706	759	27·5499546	9·121801
648	25·4558441	8·653497	704	26·5329983	8·895920	760	27·5680975	9·125805
649	25·4754784	8·657946	705	26·5518361	8·900130	761	27·5862284	9·129806
650	25·4950976	8·662391	706	26·5706605	8·904336	762	27·6043475	9·133803
651	25·5147016	8·666831	707	26·5894716	8·908538	763	27·6224546	9·137797
652	25·5342907	8·671266	708	26·6082694	8·912737	764	27·6405499	9·141788
653	25·5538647	8·675697	709	26·6270539	8·916931	765	27·6586334	9·145774
654	25·5734237	8·680124	710	26·6458252	8·921121	766	27·6767050	9·149757
655	25·5929678	8·684546	711	26·6645833	8·925308	767	27·6947648	9·153737
656	25·6124969	8·688963	712	26·6833281	8·929490	768	27·7128129	9·157714
657	25·6320112	8·693376	713	26·7020598	8·933668	769	27·7308492	9·161686
658	25·6515107	8·697784	714	26·7207784	8·937843	770	27·7488739	9·165656
659	25·6709953	8·702188	715	26·7394839	8·942014	771	27·7668868	9·169622
660	25·6904652	8·706587	716	26·7581763	8·946181	772	27·7848880	9·173585
661	25·7099203	8·710983	717	26·7768557	8·950344	773	27·8028775	9·177544
662	25·7293607	8·715373	718	26·7955220	8·954503	774	27·8208555	9·181500
663	25·7487864	8·719759	719	26·8141754	8·958658	775	27·8388218	9·185453
664	25·7681975	8·724141	720	26·8328157	8·962809	776	27·8567766	9·189402
665	25·7875939	8·728518	721	26·8514432	8·966957	777	27·8747197	9·193347
666	25·8069758	8·732892	722	26·8700577	8·971101	778	27·8926514	9·197289
667	25·8263431	8·737260	723	26·8886593	8·975240	779	27·9105715	9·201229
668	25·8456960	8·741624	724	26·9072481	8·979376	780	27·9284801	9·205164
669	25·8650343	8·745985	725	26·9258240	8·983509	781	27·9463772	9·209096
670	25·8843582	8·750340	726	26·9443872	8·987637	782	27·9642629	9·213025
671	25·9036677	8·754691	727	26·9629375	8·991762	783	27·9821372	9·216950
672	25·9229628	8·759038	728	26·9814751	8·995883	784	28·0000000	9·220873
673	25·9422435	8·763381	729	27·0000000	9·000000	785	28·0178515	9·224791
674	25·9615100	8·767719	730	27·0185122	9·004113	786	28·0356915	9·228707
675	25·9807621	8·772053	731	27·0370117	9·008223	787	28·0535203	9·232619
676	26·0000000	8·776383	732	27·0554985	9·012329	788	28·0713377	9·236528
677	26·0192237	8·780708	733	27·0739727	9·016431	789	28·0891438	9·240433

No.	Sq. Root.	Cube Root.	No.	Sq. Root.	Cube Root.	No.	Sq. Root.	Cube Root.
790	28·1069386	9·244335	846	29·0860791	9·457800	902	30·0333148	9·662040
791	28·1247222	9·248234	847	29·1032644	9·461525	903	30·0499584	9·665609
792	28·1424946	9·252130	848	29·1204396	9·465247	904	30·0665928	9·669176
793	28·1602557	9·256022	849	29·1376046	9·468966	905	30·0832179	9·672740
794	28·1780056	9·259911	850	29·1547595	9·472682	906	30·0998339	9·676302
795	28·1957444	9·263797	851	29·1719043	9·476395	907	30·1164407	9·679860
796	28·2134720	9·267680	852	29·1890390	9·480106	908	30·1330383	9·683416
797	28·2311884	9·271559	853	29·2061637	9·483813	909	30·1496269	9·686970
798	28·2488938	9·275435	854	29·2232784	9·487518	910	30·1662063	9·690521
799	28·2665881	9·279308	855	29·2403830	9·491220	911	30·1827765	9·694069
800	28·2842712	9·283178	856	29·2574777	9·494919	912	30·1993377	9·697615
801	28·3019434	9·287044	857	29·2745623	9·498615	913	30·2158899	9·701158
802	28·3196045	9·290907	858	29·2916370	9·502308	914	30·2324329	9·704699
803	28·3372546	9·294767	859	29·3087018	9·505998	915	30·2489669	9·708237
804	28·3548938	9·298624	860	29·3257566	9·509685	916	30·2654919	9·711772
805	28·3725219	9·302477	861	29·3428015	9·513370	917	30·2820079	9·715305
806	28·3901391	9·306328	862	29·3598365	9·517051	918	30·2985148	9·718835
807	28·4077454	9·310175	863	29·3768616	9·520730	919	30·3150128	9·722363
808	28·4253408	9·314019	864	29·3938769	9·524406	920	30·3315018	9·725888
809	28·4429253	9·317860	865	29·4108823	9·528079	921	30·3479818	9·729411
810	28·4604989	9·321697	866	29·4278779	9·531749	922	30·3644529	9·732931
811	28·4780617	9·325532	867	29·4448637	9·535417	923	30·3809151	9·736448
812	28·4956137	9·329363	868	29·4618397	9·539082	924	30·3973683	9·739963
813	28·5131549	9·333192	869	29·4788059	9·542744	925	30·4138127	9·743476
814	28·5306852	9·337017	870	29·4957624	9·546403	926	30·4302481	9·746986
815	28·5482048	9·340838	871	29·5127091	9·550059	927	30·4466747	9·750493
816	28·5657137	9·344657	872	29·5296461	9·553712	928	30·4630924	9·753998
817	28·5832119	9·348473	873	29·5465734	9·557363	929	30·4795013	9·757500
818	28·6006993	9·352286	874	29·5634910	9·561011	930	30·4959014	9·761000
819	28·6181760	9·356095	875	29·5803989	9·564656	931	30·5122926	9·764497
820	28·6356421	9·359902	876	29·5972972	9·568298	932	30·5286750	9·767992
821	28·6530976	9·363705	877	29·6141858	9·571938	933	30·5450487	9·771484
822	28·6705424	9·367505	878	29·6310648	9·575574	934	30·5614136	9·774974
823	28·6879766	9·371302	879	29·6479325	9·579208	935	30·5777697	9·778462
824	28·7054002	9·375096	880	29·6647939	9·582840	936	30·5941171	9·781946
825	28·7228132	9·378887	881	29·6816442	9·586468	937	30·6104557	9·785429
826	28·7402157	9·382675	882	29·6984848	9·590094	938	30·6267857	9·788909
827	28·7576077	9·386460	883	29·7153159	9·593716	939	30·6431069	9·792386
828	28·7749891	9·390242	884	29·7321375	9·597337	940	30·6594194	9·795861
829	28·7923601	9·394020	885	29·7489496	9·600955	941	30·6757233	9·799334
830	28·8097206	9·397796	886	29·7657521	9·604570	942	30·6920185	9·802804
831	28·8270706	9·401569	887	29·7825452	9·608182	943	30·7083051	9·806271
832	28·8444102	9·405339	888	29·7993289	9·611791	944	30·7245830	9·809736
833	28·8617394	9·409105	889	29·8161030	9·615398	945	30·7408523	9·813199
834	28·8790582	9·412869	890	29·8328678	9·619002	946	30·7571130	9·816659
835	28·8963666	9·416630	891	29·8496231	9·622603	947	30·7733651	9·820117
836	28·9136646	9·420387	892	29·8663690	9·626201	948	30·7896086	9·823572
837	28·9309523	9·424142	893	29·8831056	9·629797	949	30·8058436	9·827025
838	28·9482297	9·427894	894	29·8998328	9·633390	950	30·8220700	9·830476
839	28·9654967	9·431642	895	29·9165506	9·636981	951	30·8382879	9·833924
840	28·9827535	9·435388	896	29·9332591	9·640569	952	30·8544972	9·837369
841	29·0000000	9·439131	897	29·9499583	9·644154	953	30·8706981	9·840813
842	29·0172363	9·442870	898	29·9666481	9·647737	954	30·8868904	9·844254
843	29·0344623	9·446607	899	29·9833287	9·651317	955	30·9030743	9·847692
844	29·0516781	9·450341	900	30·0000000	9·654894	956	30·9192497	9·851128
845	29·0688837	9·454072	901	30·0166620	9·658468	957	30·9354166	9·854562

No.	Sq. Root.	Cube Root.	No.	Sq. Root.	Cube Root.	No.	Sq. Root.	Cube Root.
958	30·9515751	9·857993	973	31·1929479	9·909178	987	31·4165561	9·956477
959	30·9677251	9·861422	974	31·2089731	9·912571	988	31·4324673	9·959839
960	30·9838668	9·864848	975	31·2249900	9·915962	989	31·4483704	9·963198
961	31·0000000	9·868272	976	31·2409987	9·919351	990	31·4642654	9·966555
962	31·0161248	9·871694	977	31·2569992	9·922738	991	31·4801525	9·969909
963	31·0322413	9·875113	978	31·2729915	9·926122	992	31·4960315	9·973262
964	31·0483494	9·878530	979	31·2889757	9·929504	993	31·5119025	9·976612
965	31·0644491	9·881945	980	31·3049517	9·932884	994	31·5277655	9·979960
966	31·0805405	9·885357	981	31·3209195	9·936261	995	31·5436206	9·983305
967	31·0966236	9·888767	982	31·3368792	9·939636	996	31·5594677	9·986649
968	31·1126984	9·892175	983	31·3528308	9·943009	997	31·5753068	9·989990
969	31·1287648	9·895580	984	31·3687743	9·946380	998	31·5911380	9·993329
970	31·1448230	9·898983	985	31·3847097	9·949748	999	31·6069613	9·996666
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